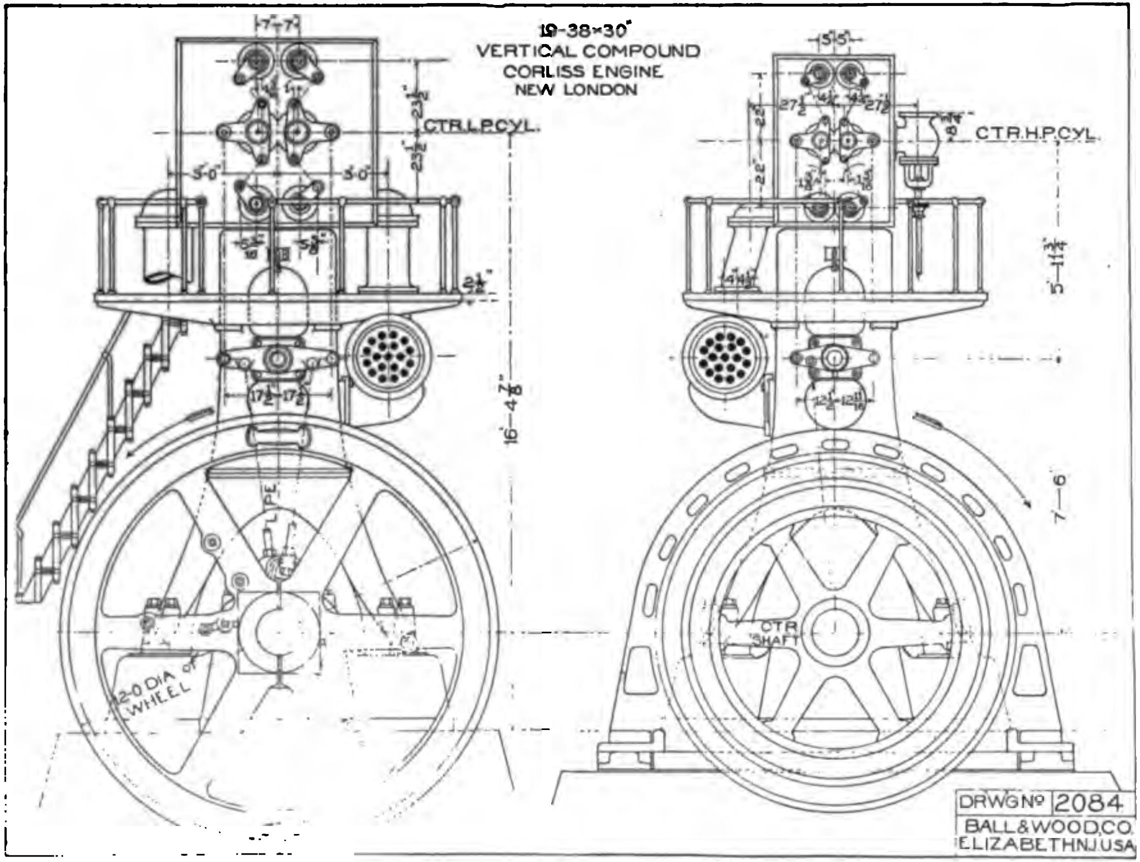

DRAWING OF VERTICAL COMPOUND CORLISS ENGINE

Reproduced from a blue print

Machine Design

A Manual of

PRACTICAL INSTRUCTION IN THE ART OF CREATING MACHINERY FOR SPECIFIC PURPOSES, INCLUDING MANY WORKING HINTS ESSENTIAL TO EFFICIENCY IN THE OPERATION AND CARE OF MACHINES, AND INCREASE OF OUTPUT

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Foreword

IN recent years, such marvelous advances have been made in the engineering and scientific fields, and so rapid has been the evolution of mechanical and constructive processes and methods, that a distinct need has been created for a series of *practical working guides*, of convenient size and low cost, embodying the accumulated results of experience and the most approved modern practice along a great variety of lines. To fill this acknowledged need, is the special purpose of the series of handbooks to which this volume belongs.

¶ In the preparation of this series, it has been the aim of the publishers to lay special stress on the *practical* side of each subject, as distinguished from mere theoretical or academic discussion. Each volume is written by a well-known expert of acknowledged authority in his special line, and is based on a most careful study of practical needs and up-to-date methods as developed under the conditions of actual practice in the field, the shop, the mill, the power house, the drafting room, the engine room, etc.

¶ These volumes are especially adapted for purposes of self-instruction and home study. The utmost care has been used to bring the treatment of each subject within the range of the com-

mon understanding, so that the work will appeal not only to the technically trained expert, but also to the beginner and the self-taught practical man who wishes to keep abreast of modern progress. The language is simple and clear; heavy technical terms and the formulæ of the higher mathematics have been avoided, yet without sacrificing any of the requirements of practical instruction; the arrangement of matter is such as to carry the reader along by easy steps to complete mastery of each subject; frequent examples for practice are given, to enable the reader to test his knowledge and make it a permanent possession; and the illustrations are selected with the greatest care to supplement and make clear the references in the text.

¶ The method adopted in the preparation of these volumes is that which the American School of Correspondence has developed and employed so successfully for many years. It is not an experiment, but has stood the severest of all tests—that of practical use—which has demonstrated it to be the best method yet devised for the education of the busy working man.

¶ For purposes of ready reference and timely information when needed, it is believed that this series of handbooks will be found to meet every requirement.

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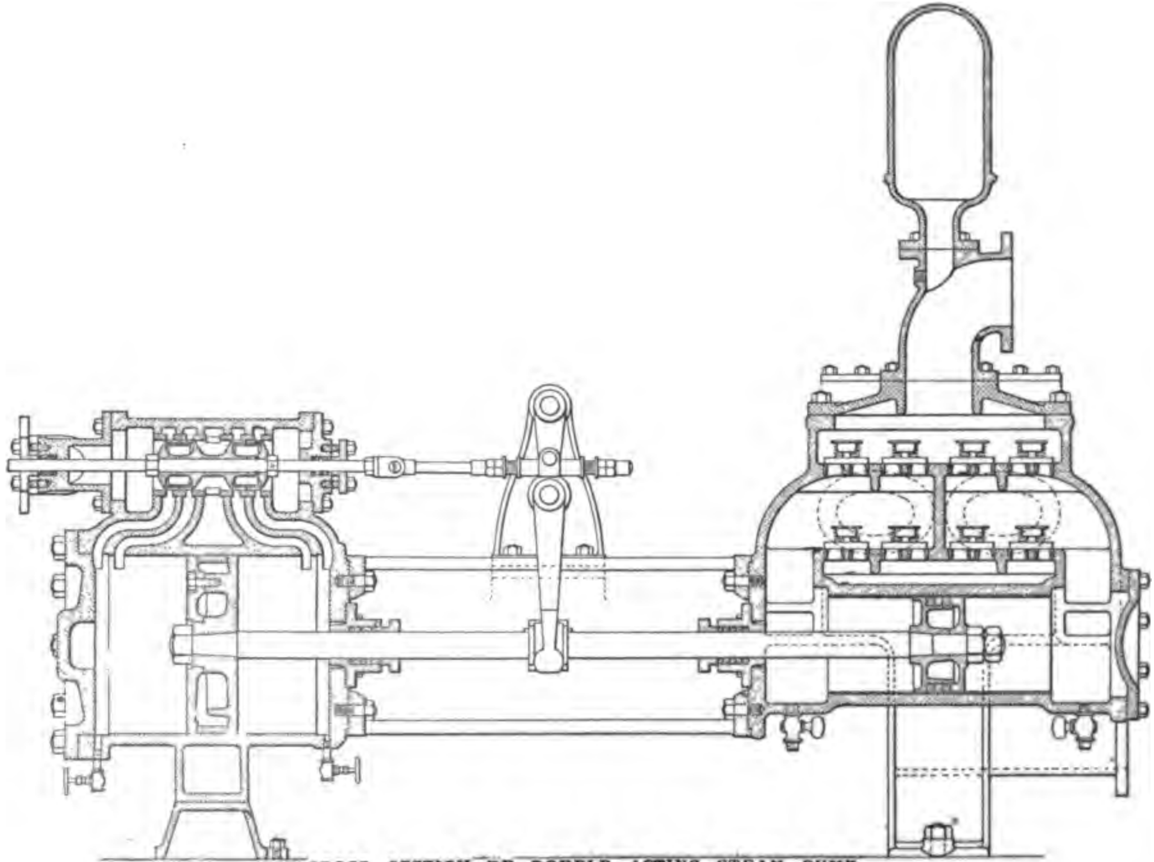


CROSS SECTION OF DOUBLE ACTING STEAM PUMP.

MACHINE DESIGN.

PART I.

Definition. Machine Design is the art of mechanical thought development, and specification.

It is an art, in that its routine processes can be analyzed and systematically applied. Proficiency in the art positively cannot be attained by any "short cut" method. There is nothing of a spectacular nature in the methods of Machine Design. Large results cannot be accomplished at a single bound, and success is possible only by a patient, step-by-step advance in accordance with well-established principles.

"Mechanical thought" means the thinking of things strictly from their mechanical side; a study of their mechanical theory, structure, production, and use; a consideration of their mechanical fitness as parts of a machine.

"Mechanical development" signifies the taking of an idea in the rough, in the crude form, for example, in which it comes from the inventor, working it out in detail, and refining and fixing it in shape by the designing process. Ideas in this way may become commercially practicable designs.

"Mechanical specification" implies the detailed description of designs, in such exact form that the shop workmen are enabled to construct completely and put in operation the machines represented in the designs.

The object of Machine Design is the creation of machinery for specific purposes. Every department of a manufacturing plant is a controlling factor in the design and production of the machines built there. A successful design cannot be out of harmony with the organized methods of production. Hence in the high development of the art of Machine Design is involved a knowledge of the operations in all the departments of a manufacturing plant. The student is therefore urged not only to familiarize himself with the direct production of machinery, but to study the relation thereto of the allied commercial departments.

He should get into the spirit of business at the start, get into the shop atmosphere, execute his work just as though the resulting design were to be built and sold in competition. He should visit shops, work in them if possible, and observe details of design and methods of finishing machine parts. In this way he will begin to store up bits of information, practical and commercial, which will have valuable bearing on his engineering study.

The labor involved in the design of a complicated automatic machine is evidenced by the designer's wonderful familiarity with its every detail as he stands before the completed machine in operation and explains its movements to an observer. The intricate mass of levers, shafts, pulleys, gears, cams, clutches, etc., etc., packed into a small space, and confusing even to a mechanical mind, seems like a printed book to the designer of them.

This is so because it is a familiar journey for the designer's mind to run over a path which it has already traversed so many times that he can see every inch of it with his eyes shut. Every detail of that machine has been picked from a score or more of possible ideas. One by one, ideas have been worked out, laid aside, and others taken up. Little by little, the special fitness of certain devices has become established, but only by patient, careful consideration of others, which at first seemed equally good.

Every line, and corner, and surface of each piece, however small that piece may be, has been through the refining process of theoretical, practical, and commercial design. Every piece has been followed in the mind's eye of its designer from the crude material of which it is made, through the various processes of finishing, to its final location in the completed machine; thus its bodily existence there is but the realization of an old and familiar picture.

What wonder that the machine seems simple to the designer of it! As he looks back to the multitude of ideas invented, worked out, considered and discarded, the machine in its final form is but a trifle. It merely represents a survival of the fittest.

No successful machine, however simple, was ever designed that did not go through this slow process of evolution. No machine ever just simply happened by accident to do the work for which it is valued. No other principle upon which the suc-

cessful design of machinery depends is so important as this careful, patient consideration of detail. A machine is seldom unsuccessful because some main point of construction is wrong. The principal features of a machine are usually the easiest to determine. It is a failure because some little detail was overlooked, or hastily considered, or allowed to be neglected, because of the irksome labor necessary to work it out properly.

There is no task so tedious, for example, as the devising of the method of lubricating the parts of a complicated machine. Yet there is no point of design so vital to its life and operation as an absolute assurance of an adequate supply of oil for the moving parts at all times and under all circumstances. Suitable means often cannot be found, after the parts are together, hence the machine goes into service on a risky basis, with the result, perhaps, of early failure, due to "running dry." Good designers will not permit a design to leave their hands which does not provide practically automatic oiling, or at least such means of lubrication that the operator can offer no excuse for neglecting to oil his machine. This is but a single illustration of many which might be presented to impress the definite and detail character necessary in work in Machine Design.

Relation. The relation which Machine Design should correctly bear to the problems that it seeks to solve, is twofold; and there are, likewise, two points of view corresponding to this twofold relation, from which a study of the subject should be traced. Neither of these can be discarded and an efficient mastery of the art attained. These points are—

I. Theory.

II. Production.

I. *Theory.* From this point of view, Machine Design is merely a skeleton or framework process, resulting in a representation of ideas of pure motion, fundamental shape, and ideal proportion. It implies a working knowledge of physical and mathematical laws. It is a strictly scientific solution of the problem at hand, and may be based purely on theory which has been reasoned out by calculation or deduced from experiment. This is the only sure foundation for intelligent design of any sort.

But it is not enough to view the subject from the standpoint

of theory alone. If we stopped here we should have nothing but mechanisms, mere laboratory machines, simply structures of ingenuity and examples of fine mechanical skill. A machine may be correct in the theory of its motions; it may be correct in the theoretical proportions of its parts; it may even be correct in its operation for the time being; and yet its complication, its misdirected and wasteful effort, its lack of adjustment, its expensive and irregular construction, its lack of compactness, its difficulty of ready repair, its inability to hold its own in competition—any of these may throw the balance to the side of failure. Such a machine, commercially considered, is of little value. No shop will build it, no machinery house will sell it, nobody will buy it if it is put on the market.

Thus we see that, aside from the theoretical correctness of principle, the design of a machine must satisfy certain other exacting requirements of a distinctly business nature.

II. *Production.* From this point of view, Machine Design is the practical, marketable development of mechanical ideas. Viewed thus, the theoretical, skeleton design must be so clothed and shaped that its production may be cheap, involving simple and efficient processes of manufacture. It must be judged by the latest shop methods for exact and maximum output. It must possess all the good points of its competitor, and, withal, some novel and valuable ones of its own. In these days of keen competition it is only by carefully studied, well-directed effort toward rapid, efficient, and, therefore, cheap production that any machine can be brought to a commercial basis, no matter what its other merits may be. All this must be thought of and planned for in the design, and the final shapes arrived at are quite as much a result of this second point of view as of the first.

As a good illustration of this, may be cited the effect of the present somewhat remarkable development of the so-called "high speed" steels. The speeds and feeds possible with tools made of these steels are such that the driving power, gearing, and feed mechanism of the ordinary lathe are wholly inadequate to the demands made upon them when working the tool to its limit. This means that the basis of design as used for the ordinary tool steel will not do, if the machine is expected to stand up to the

cuts possible with the new steels. Hence, while the old designs were right for the old standard, a new one has been set, and a thorough revision on a high-speed basis is imminent, else the market for them as machines of maximum output will be lost.

From these definitions it is evident that the designer must not only use all the theory at his command, but must continually inform himself on all processes and conditions of manufacture, and keep an eye on the tendency of the sales markets, both of raw material and the finished machinery product. This is what in the broadest sense is meant by the term "Mechanical Thought," thought which is directed and controlled, not only by theoretical principle but by closely observed practice. From the feeblest pretenders of design to those engineers who consummate the boldest feats and control the largest enterprises, the process which produces results is always the same. Although experience is necessary for the best mechanical judgment, yet the student must at least begin to cultivate good mechanical sense very early in his study of design.

Invention. Invention is closely related to Machine Design, but is not design itself. Whatever is invented has yet to be designed. An invention is of little value until it has been refined by the process of design.

Original design is of an inventive nature, but is not strictly invention. Invention is usually considered as the result of genius, and is announced in a flash of brilliancy. We see only the flash, but behind the flash is a long course of the most concentrated brain effort. Inventions are not spontaneous, are not thrown off like sparks from the blacksmith's anvil, but are the result of hard and applied thinking. This is worth noting carefully, for the same effort which produces original design may develop a valuable invention. But there is little possibility of inventing anything except through exhaustive analysis and a clear interpretation of such analysis.

Handbooks and Empirical Data. The subject matter in these is often contradictory in its nature, but valuable nevertheless. Empirical data are data for certain fixed conditions and are not general. Hence, when handbook data are applied to some specific case of design, while the information should be used in the freest

manner, yet it must not be forgotten that the case at hand is probably different, in some degree, from that upon which the data were based, and unlike any other case which ever existed or will ever again exist. Therefore the data should be applied with the greatest discretion, and when so applied will contribute to the success of the design at least as a check, if not as a positive factor.

The student should at the outset purchase one good handbook, and acquire the habit of consulting it on all occasions, checking and comparing his own calculations and designs therefrom. Care must be taken not to become tied to a handbook to such an extent that one's own results are wholly subordinated to it. Independence in design must be cultivated, and the student should not sacrifice his calculated results until they can be shown to be false or based on false assumption. Originality and confidence in design will be the result if this course be honestly pursued.

Calculations, Notes, and Records. Accurate calculations are the basis of correct proportions of machine parts. There is a right way to make calculations and a wrong way, and the student will usually take the wrong way unless he is cautioned at the start.

The wrong way of making calculations is the loose and shiftless fashion of scratching upon a scrap of detached paper marks and figures, arranged in haphazard form, and disconnected and incomplete. These calculations are in a few moments' time totally meaningless, even to the author of them himself, and are so easily lost or mislaid that when wanted they usually cannot be found.

Engineering calculations should always be made systematically, neatly, and in perfectly legible form, in some permanently bound blank book, so that reference may always be had to them at any future time for the purpose of checking or reviewing. Put all the data down. Do not leave in doubt the exact conditions under which the calculations were made. Note the date of calculation.

If a mistake in figures is made, or a change is found necessary, never rub out the figures or tear out the leaf, or in any way obliterate the figures. Simply draw a bold cross through the wrong part and begin again. Often a calculation which is supposed to be wrong is later shown to be right, or the facts which caused the error may be needed for investigation and comparison. Time which

is spent in making figures is always valuable time, time too precious to be thrown away by destroying the record.

The recording of calculations in a permanent form, as just described, is the general practice in all modern engineering offices. This plan has been established purely as a business policy. In case of error it locates responsibility and settles dispute. Consistent designing is made possible through the records of past designs. Proposals, estimates, and bids may often be made instantly, on the basis of what these record books show of sizes and weights. This bookkeeping of calculations is as important a factor of systematic engineering as bookkeeping of business accounts is of financial success.

The student should procure for this purpose a good blank book with a firm binding, size of page not smaller than 6 by 8 inches (perhaps 8 by 11 inches may be better), and every calculation, however small and apparently unimportant, should be made in it.

Sample pages of engineering calculations are reproduced in Figs. 3 to 9. Note the sketch showing the forces. Note the clear statement of data. Note the systematic writing of the equations, and the definite substitutions therein. Note the heavy double underscoring of the result, when obtained. There is nothing in the whole process of the calculation that cannot be reviewed at any moment by anybody, and in the briefest time.

The development of a personal note-book is of great value to the designer of machinery. The facts of observation and experience recorded in proper form, bearing the imprint of intimate personal contact with the points recorded, cannot be equalled in value by those of any hand or reference book made by another. There is always a flavor about a personal note-book, a sort of guarantee, which makes the use of it by its author definite and sure.

The habit of taking and recording notes, or even knowing what notes to take, is an art in itself, and the student should begin early to make his note-book. Aside from the value of the notes themselves as a part of his personal equipment, the facility with which his eye will be trained to see and record mechanical things will be of great value in all of his study and work. How many men go through a shop and really see nothing of the opera-

tions going on therein, or, seeing them, remember nothing! An engineer, trained in this respect, will to a surprising degree be able to retain and sketch little details which fall under his eye for a brief moment only, while he is passing through a crowded shop.

Some draftsmen have the habit of copying all the standard tables of the various offices in which they work. While these are of some value in a few cases, yet this is not what is meant by a good note-book in the best sense. Ideas make a good note-book, not a mere tabulation of figures. If the basis upon which standards are founded can be transferred to permanent personal record, or novel methods of calculation, or simple features of construction, or data of mechanical tests, or efficient arrangement of machinery—if *these* can be preserved for reference, the note-book will be of greatest value.

Whatever is noted down, make clear and intelligible, illustrating by a sketch if possible. Make the note so clear that reference to it after a long space of years would bring the whole subject before the mind in an instant. If this is not done the author of the note himself will not have patience to dig out the meaning when it is needed; and the note will be of no value.

METHOD OF DESIGN.

The fundamental lines of thought and action which every designer follows in the solution of any problem in any class of work whatsoever, are four in number. The expert may carry all these in mind at the same time, without definite separation into a a step-by-step process; but the student must master them in their proper sequence, and thoroughly understand their application. In these four are concentrated the entire art of Machine Design. When they have become so familiar as to be instinctively applied on any and all occasions, good design is the result. The only other quality which will facilitate still further the design of good machinery is experience; and that cannot be taught, it must be acquired by actual work.

1. Analysis of Conditions and Forces. First, take a good square look at the problem to be solved. Study it from all sides, view it in all lights, note the worst conditions which can possibly exist, note the average conditions of service, note any special or irregular service likely to be called for.

With these conditions well in mind, make a careful analysis of all the forces, maximum as well as average, which may be brought into play. Make a rough sketch of the piece under consideration, and put in these forces. Be sure that these forces are at least approximately right. Go over the analysis carefully again and again. Remember that time saved at the beginning by hasty and poor analysis will actually be time lost at the end; and if the machine actually fails from this reason, heavy financial loss in material and labor will occur. Any haste toward completion of the structure beyond the roughest outline, without this careful study of forces, is a blind leap in the dark, entirely unscientific, and almost certain to result in ultimate failure.

On the other hand this principle may be carried too far. In trying to make the analysis thorough and the forces accurate, it is quite possible to consume more than a reasonable amount of time. Again, it is not always easy, and frequently impossible, to determine exactly the forces acting on a given piece. But their *nature*, whether sudden or slowly applied, rapid in action or only occurring at intervals, and their *approximate* direction and magnitude at least, are always capable of analysis. There are few, if any, cases where close assumptions cannot be made on the above basis and the design proceeded with accordingly. Hence the danger of too great refinement of analysis is simply to be avoided by the designer's plain business sense.

The first tendency of the student is to pass over the study of the forces as dull and dry, and attempt the design at once. He soon finds himself facing problems of which he sees no possible solution, and he bases his design on pure guess-work. This is the only solution possible from such a point of view, and is really no solution at all. A guess which has some rational backing is often successful; but in that case some analysis is required, and it is not a pure guess, but falls under the very principle we are considering.

There is no short cut to the design of machine parts which avoids this full understanding of the forces that they must sustain. The size of a belt depends upon the maximum pull upon it, and the designing of belts is nothing but providing sufficient cross-section of leather to prevent the belt tearing under

the pull. Again, if pulley arms are not to break, or shafts twist off, or bolts be torn apart, or the teeth of gears fail, or keys and pins shear off, we must first, of course, find out what forces exist which are likely to produce stress that may lead to such breakage. We should not guess at the sizes, and then run the machine to see if breakage results, and then guess again. Machines are sometimes built in this way, but it is an unreasonable and uncertain method. We must use every effort to foresee the stress which a piece is liable to receive, before we decide its size. We must know all the forces approximately, if not positively. The analysis must be thorough enough to permit of reasonable assumption, if not positive assertion. It is manifestly impossible to solve any problem until we know exactly what the problem is; and a full analysis *is* the statement of the problem.

2. **Theoretical Design.** After we know by careful analysis what stress the machine part has to sustain, the next step is so to design it that it will theoretically resist the applied forces with the least expenditure of material.

We often see machinery with the metal of which it is made distributed in the worst possible manner. In places where the stress is heavy and a rigid member is needed, we find a weak, springy part; while in other parts, where there are no forces to be resisted, or vibration to be absorbed, there seems to be a waste of good material. Whether in such case the analysis of the forces was poor, or perhaps not made at all, or whether a knowledge of how to design so as to resist the given forces was wholly absent, cannot be told. At any rate, lack of either or both is clearly shown in the result.

Any member of a machine may vary in form from a solid block or chunk of material to an open ribbed structure. The solid chunk fills the requirement as far as strength is concerned, unless it is so heavy as to fail from its own weight. But such construction is poor design, except in cases where the concentration of heavy mass is necessary to absorb repeated blows like those of a hammer. The possibility of these blows should, however, have been determined in the analysis; and the solid, anvil construction then becomes theoretical design for that analysis.

For steadily applied loads an open, ribbed, or hollow box

structure can be made which will distribute the metal where it is theoretically needed, and each fiber will then sustain its proper share of the load. In this way weight, cost, and appearance are heeded; and the service of the piece is as good as, and probably better than, it would be with the clumsy, solid form.

There is no such thing as putting too much theory into the design of machinery. The strongest trait which an engineer can have is absolute faith in his analysis and calculations, and their reproduction in his theoretical design. Theoretical design is an indication of scientific advance in the art, and some of the greatest steps of progress which have been made in recent years have been accomplished through a purely theoretical study of machine structure.

It will never do, however, to be satisfied with theoretical design when it is not in accord with modern commercial and manufacturing considerations. Hence the next step after the determination of the theoretical design is the study of it from the producing standpoint.

3. **Practical Modification.** All theoretical design viewed from the business standpoint is worthless, unless it has been subjected to the test of cheap and efficient production. Each machine detail, though correct in theory, may yet be improperly shaped and unfit for the part it is to play in the general scheme of manufacture.

The conditions here involved are changeable. What is good design in this decade may be bad in the next. In this light the designer must be a close student of the signs of the times; he must follow the march of progress, closely applying existing resources, conditions, and facilities, otherwise he cannot produce up-to-date designs. The introduction of new raw materials, the cheapening of production of others, the changing of shop methods, the use of special machinery, the opening of new markets, the development of new motive agents, — all these and many others are constantly demanding some modification in design to meet competition.

Illustrative of this, note the change which has been wrought by the development of electric power, the rise and decline of the bicycle business, the present manufacture of automobiles, the last named especially with reference to the development of the small motive unit, the gasolene engine, the steam engine, etc. The

design of much machinery has been materially changed to meet the exacting demands of these new enterprises.

Practical modifications of design necessary to meet the limitations of construction in the pattern shop, foundry, and machine shop are of daily application in the designer's work. He must keep in his mind's eye at all times the workmen and the processes they use to create his designs in metal in the shop.

"How can this be made?" "Can it be made at all?" "Can it be made cheaply?" "Will it be simple in operation after it is made?" "Can it be readily removed for repair?" "Can it be lubricated?" "How can it be put in place?" "How can it be gotten out?" "Will it be made in small quantities or large?" "Will it sell as a special or standard machine?" etc., etc.

The consideration of such questions as these is a practical necessity as a business matter. No other feature affects the design of machinery more, perhaps; for designs which cannot be built as business propositions are no designs at all.

The student, it is true, may not have the extended shop knowledge which is essential to this; but he can do much for himself by visiting shops whenever possible, getting hold of shop ways of doing things, and invariably treating his work as a business matter. Though a man may not be a pattern maker, molder, blacksmith, or machinist, yet he can soon gain ideas of the processes in each of these branches which will be of immense advantage to him in his designing work.

4. Delineation and Specification. This means the clear and concise representation of the design by mechanical drawings.

This is as much a part of the routine method of Machine Design as the other three points which have been discussed. The mere act of putting the results of mechanical thinking on paper is one of the greatest helps to force thinking machinery to systematic and definite action. A designer never thinks very long without drawing something, and the student must bring himself to feel that a drawing in its first sense is a means of helping his own thought, and must freely use it as such.

In its second and final sense, the drawing is an order and specification sheet from the designer to the workman. Design

which stops short of exact, finished delineation in the form of working shop drawings is only half done. In fact the possibility of a piece being thus exactly drawn is often the crucial test of its feasibility as a part of a machine. It is easy to make general outlines, but it is not so easy to get down to finished detail. It is safe to say that there is no one thing productive of more trouble, delay and embarrassment, and waste of time and money in the shop, when there need be none from this cause, than a poor detail drawing. The efficiency of the process of design is not fully realized, and failures are often recorded where there should be success, merely because the indefiniteness permitted by the designer in the drawings naturally transmitted itself to the workman, and he in turn produced a part indefinite in form and operation.

The actual process of drawing in the development of a design may be outlined as follows :

Rough sketches merely representing ideas, not drawn to scale, are first made. These are of use only so far as the choice of mechanical ideas is concerned, and to carry preliminary dimensions.

Following these sketches, comes a layout to scale, of the favored sketch, a working out of the relative sizes and location of the parts. This drawing may be of a sketchy nature, carrying a principal dimension here and there to fix and control the detailed design. In this drawing the design is developed and general detail worked out. The minute detail of the individual parts is, however, left to the subsequent working drawing.

This layout drawing may now be turned over to an expert draftsman or detail designer, who picks out each part, makes an exact drawing of it, studying every little detail of its shape, and finally adds complete dimensions and specifications so that the workman is positively informed as to every point of its construction.

General drawings and cross sections constitute the last step in the process of complete delineation. These show the parts assembled in the complete machine. They also serve a valuable purpose to the draftsman in checking up the dimensions of the detail drawings. Errors which have escaped previous notice are often discovered in this way. The layout, mentioned above, is sometimes finished up into a general drawing; but it is safer to make an entirely new drawing, as changes in detail are often necessary after the layout is made.

The four fundamental lines of thought and action noted above may be summarized thus—"**analyze and theorize, modify and delineate.**" This is a maxim easy to remember, applicable to every problem in Machine Design, and always provides the answer to the question "What shall I do, how shall I proceed?" by pointing out the proper sequence in the course to be followed.

CONSTRUCTIVE MECHANICS.

Mechanics is a constructive science, its principles lying at the root of the design and operation of all machinery. It is usually taught, however, as an advanced mathematical subject; and the student gets his original conceptions of forces, moments, and beams in the abstract, before he realizes the constructive value of such conceptions. By "Constructive Mechanics" is meant the study of a machine purely from its constructive side, the viewing of the parts with respect to their "mechanics," and satisfying the requirements of the same in form and arrangement.

The student may cultivate this habit of clear, mechanical perception by constantly noting the "mechanics" of the simple structures which he sees in his daily routine of work. Aside from machinery, in which the "mechanics" is often obscure, the world is full of simple examples of natural strength and symmetry, explainable by application of the principles of pure "mechanics."

Posts and pillars are largest at their bases; overhanging brackets or arms are spread out at the fastening to the wall; heavy swinging gates are counter-balanced by a ponderous weight; the old-fashioned well sweep carries its tray of stones at the end, adjusting the balance to a nicety; these are examples of things depending for their form and operation upon the principles of "mechanics." The building of them involved "constructive mechanics," and yet their constructor perhaps never heard of the science, using merely his natural sense of mechanical fitness Such simple reasoning is, however, Constructive Mechanics.

Forces, Moments, and Beams. Machines are nothing but a collection of (1) parts taking direct stress, or (2) parts acting as loaded beams. Forces acting *without* leverage produce direct stress on the sustaining part. Forces acting *with* leverage pro-

duce a moment; the sustaining member is a beam, and the stress therein depends on the theory of beams, as explained in "Mechanics."

An example of the first is the load on a rope, the force acting without leverage, and the rope therefore having a direct stress put upon it.

An example of the second is a push of the hand on the crank of a grindstone. A moment is produced about the hub of the crank; the arm of the crank is a beam, and the stress at any point of it may be found by the method of theory of beams.

Tension, Compression, and Torsion. The stress induced in the sustaining part, whether tensile, compressive, or torsional, is caused by the application of forces, either acting directly without leverage, or with leverage in the production of moments.

The forces applied from external sources are at constant war with the resisting forces due to the strength of the fibres of the material composing the machine members. The moments of the *external* forces are constantly exerted against and balanced by the moments of the *internal* resistance of the material. Hence, design, from a strength standpoint, is merely a balancing of internal strength against external force. In other words, we may in all cases write a sign of equality, place the applied effort on one side, the effective resistance on the other, and we shall have an equation, which, if capable of solution, will give the proper proportions of the parts considered.

External Force = Internal Resistance.

External Moment = Internal Moment of Resistance.

Expressed in terms of the "Mechanics:"

$$P = AS \qquad (1)$$

$$B \text{ or } T = \frac{SI}{c} \qquad (2)$$

In these formulas, which are perfectly general,

P=direct load in pounds.

A=area of effective material, in square inches.

S=working fibre stress of the material (tensile, compressive, or shearing), in pounds per square inch.

B or T=external moment (bending or torsional), in inch-pounds.

I=moment of inertia (direct or polar), of the resisting section.

c=distance of the most remote fibre of the resisting section from the neutral axis.

P may produce direct tensile, compressive, or shearing stress.

B may produce tensile or compressive stress, and requires use of direct moment of inertia in either case.

T produces shearing stress, and requires use of polar moment of inertia.

The origin of formula (1) is obvious, the assumption being that the fibre stress is equally distributed to every particle in the area "A."

The development of formula (2) is given in any text-book in Mechanics. It requires the aid of the Calculus, however. Any good handbook gives values for both the direct moment of inertia and the polar moment of inertia for quite a large variety of sections, so that further reference is an easy matter for the student. These values are also obtained through the methods of the Calculus.

The reason for introducing these formulas at this time is to call the attention of the student especially to the fact of their universal and fundamental use in all problems concerning the strength of machine parts. Nearly every computation may be reduced to or expanded from these two simple equations. Many complex combinations occur, of course, which will not permit simple and direct application of these formulas, but the student will do well to place himself in perfect command of these two. Assuming that he is able to analyze forces, and compute the simple moment at the point where he wishes to find the strength of section, the rest is the mere insertion of the assumed working fibre stress of the material in the formula (2) above, and solution for the quantity desired.

When the case is one of combined stress, the relation becomes more complicated and difficult of analysis and solution. The most common case is where bending is combined with torsion, as in the case of a shaft transmitting power, and at the same time loaded transversely between bearings. In fact there are very few cases of shafts in machines, which, at some part of their length, do not have this combined stress. In this case the method of procedure is to find the simple bending moment and the simple torsional moment separately, in the ordinary way. Then the theory of elasticity furnishes us with a formula for an equivalent bending or an equivalent torsional moment which is supposed to produce the same effect upon the fibres of the material as the combined

action of the two simple moments acting together. In other words, the separate moments combined in action, being impossible of solution in that form, are reduced to an equivalent simple moment and the solution then becomes the same as for the previous case.

These equivalent equations are given below, the subscript "e" being added to express separation from the simple moment:

$$B_e = \frac{B}{2} + \tfrac{1}{2}\sqrt{B^2+T^2} \qquad (3)$$

$$T_e = B + \sqrt{B^2+T^2} \qquad (4)$$

B_e and T_e, found from these equations, are the external moments, and are to be equated to the internal moments of resistance of the section precisely as if they were simple bending or torsional moments. Either may be used. For shafts (4) is generally used, being the simpler of the two in form.

FRICTION AND LUBRICATION.

The parts of a machine which have no relative motion with regard to each other are not dependent upon lubrication of their surfaces for the proper performance of their functions. In cases where relative motion does occur, as between a planer bed and its ways, a shaft and its bearing, or a driving screw and its nut, friction, and consequent resistance to motion, will inevitably occur. Heat will be generated, and cutting or scoring of the surfaces will take place if the surfaces are allowed to run together dry.

This difficulty, which exists with all materials, cannot be overcome, for it is a result of roughness of surface, characteristic of the material even when highly finished. The problem of the designer, then, is to take conditions as he finds them, and, as he cannot change the physical characteristics of materials, so choose those which are to rub together in the operation of the machine that friction will be reduced to the lowest possible limit. Now it fortunately happens that there are certain agents like oil and graphite, which seem to fill up the hollows in the surface of a solid material, and which themselves have very little friction on other substances. Hence, if a machine permits by its design an automatic supply of these lubricating agents to all surfaces having

motion between them, friction may be reduced to the lowest limit.

If this full supply of lubricant be secured, and the parts still heat and cut, then the fault may be traced to other causes, such as springy surfaces, localization of pressure, or insufficient radiating surface to carry away the heat of friction as fast as it is generated.

Lubricating agents are of a nature running from the solid graphite form to a thick grease, then to a heavy dark oil, and finally to a thin, fluid oil flowing as freely as water. The solid and heavy lubricants are applicable to heavily loaded places where the pressure would squeeze out the lighter oils. Grease, forced between the surfaces by compression grease cups, is an admirable lubricator for heavy machinery under severe service. High-speed and accurate machinery, lightly loaded, requires a thin oil, as the fits would not allow room for the heavier lubricants to find their way to the desired spot. The ideal condition in any case is to have a film of lubricant always between the surfaces in contact, and it is this condition at which the designer is always aiming in his lubricating devices.

Oil ways and channels should be direct, ample in size, readily accessible for cleaning, and distributing the oil by natural flow over the full extent of the surface. Hidden and remote bearings must be reached by pipes, the mouths of which should be clearly indicated and accessible to the operator of the machine. Such pipes must be straight, if possible, and readily cleaned.

There is one practical principle affecting the design of methods of lubrication of a machine which should be borne in mind. This is, "Neglect and carelessness by the operator *must* be provided for." It is of no use to say that the ruination of a surface or hidden bearing is due to neglect by the operator, if the means for such lubrication are not perfectly obvious. This is "locking the door after the horse is stolen." The designer has not done his duty until he has made the scheme of lubrication so plain that every part *must* receive its proper supply of oil, except by gross and willful negligence, for which there can be no possible just excuse.

WORKING STRESSES AND STRAINS.

Some persons object to the use of these terms, as one is frequently used for the other, and misunderstanding results. This

is doubtless true; but the student may as well learn the true relation of the terms once for all, because he will frequently run across them in his reading and reference work, and should interpret them rightly. The strict relation of the two is as follows:

Stress is the internal force in a piece resisting the external force applied to it. A weight of ten pounds hanging on a rope produces a *stress* of ten pounds in the rope.

Strain is the change of shape, or deformation, in a piece resisting an external force applied to it. If the above weight of ten pounds stretches the rope $\frac{1}{4}$ inch, the *strain* is $\frac{1}{4}$ inch.

Unit stress is stress per unit area, e. g., per square inch.

Unit strain is strain per unit length, e. g., per inch length.

In the above case, if the rope were $\frac{1}{2}$ square inch in area and 30 inches long, the unit stress, or intensity of stress, is $10 \div \frac{1}{2} = 20$ pounds per square inch; the unit strain is $\frac{1}{4} \div 30 = \frac{1}{120}$ inch per inch.

When stress is induced in a piece, the strain is practically proportional to the stress for all values of the stress below the elastic limit of the material; and when the external load is removed the strain will entirely disappear, or the recovering power of the material will restore the piece to the original length.

Illustrating by the case above, on the supposition that the elastic limit has not been reached by the stress of 20 pounds per square inch, if the load of 10 pounds were taken off, the $\frac{1}{4}$-inch strain would disappear and the rope return to its original length; if the load were changed to $\frac{1}{2}$ of 10 pounds, or 5 pounds, the strain would be $\frac{1}{2}$ of $\frac{1}{4}$ inch, or $\frac{1}{8}$ inch.

Now it is found that if we wish a piece to last in service for a long time without danger of breakage, we must not permit it to be stressed anywhere near the elastic limit value. If we do, although it will probably not break at once, it is in a dangerous condition, and not well suited to its requirements as a machine member. The technical name for this weakening effect is "fatigue." It is further found that the fatigue due to this repeated stress is reached at a lower limit when the stress is alternating in character than when it is not. In other words, if we first pull on a piece and then push on it, we shall first have the piece in tension and then in compression; this alternation of stress repeated to

near the elastic limit of the material will fatigue it, or wear out the fibres, and it will finally fail. If, however, we first pull on the piece with the same force as before, and then let go, we shall first have the piece in tension and then entirely relieved; such repetition of stress will finally "fatigue" the material, but not so quickly as in the first case. Experiments indicate that it may take twice as many applications in the latter case as in the former.

The working stress of materials permissible in machines is based on the above facts. The breaking strength divided by a liberal factor of safety will not necessarily give a desirable working stress. The question to be answered is, "Will the assumed working fibre stress permit an indefinite number of applications of the load without fatiguing the material?"

Hence we see that the same material may be safely used under different assumptions of working stress. For example, a rotating shaft, heavily loaded between bearings. acts as a beam which in each revolution is having its particles subjected, first to a maximum tensile stress, and then to a maximum compressive stress. This is obviously a very different stress from that which the same piece would receive if it were a pin in a bridge truss. In the former we have a case where the stress on each particle reverses at each revolution, while in the latter we have merely the same stress recurring at intervals, but never becoming of the opposite character. For ordinary steel, a value of 8,000 would be reasonable in the former case, while in the latter it may be much higher with safety, perhaps nearly double.

From the facts stated above, it is evident that exact values for working fibre stress cannot be assumed with certainty and applied broadly in all cases. If the elastic limit of the material is definitely known we can base our working value quite surely on that.

With but a general knowledge of the elastic limit, ordinary steel is good for from 12,000 to 15,000 pounds per square inch non-reversing stress, and 8,000 to 10,000 reversing stress. Cast iron is such an uncertain metal on account of its variable structure that stresses are always kept low, say from 3,000 to 4,000 for non-reversing stress, and 1,500 to 2,500 for reversing stress.

With these values as a guide, and the special conditions controlling each case carefully studied, reasonable limits may be

assigned for working stress, not only of steels, various grades of cast iron, and mixtures of the same, but of other alloys, brass, bronze, etc. Gun metal, semi-steel, and bronze are intermediate in strength between cast iron and steel. Data on the strength of materials are available in any of the handbooks, and should be consulted freely by the student. They will be found somewhat conflicting, but will assist the judgment in coming to a conclusion.

Application to Practical Case. In actual practice the only information which the designer has, upon which to base his design, is the object to be accomplished. He must choose or originate suitable devices, develop the arrangement of the parts, make his own assumptions regarding the operation of the machine, then *Analyze and Theorize, Modify and Delineate* each detail as he meets it.

This, it will be found, is a very different matter from taking some familiar piece of machinery, such as a pulley, or a shaft, or a gear, as an isolated case, the load being definitely given, and proceeding with the design. This is easily done, but is only half the problem, for machine parts, such as pulleys, gears, and shafts, do not confront the designer tagged or labeled with the conditions they are to meet. He is to provide parts to meet the specific conditions, and it is as much a part of his designing method to know how to attack the design of a machine as it is to know how to design the parts in detail after the attack has reduced the members to definitely loaded structures. The whole process must be gone through, the preliminary sketches, calculations, and layout, all of which precede the detail design and working drawings; and no step of the process can be omitted.

It is for this reason that the present case used for illustration is carried out quite thoroughly. The student should make himself familiar with every step of the designing method as applied to this simple case of design. More complex problems, handled in the same way, will simplify themselves; and when the point is reached where confidence exists to take hold of the design of any machine, however unfamiliar its object may be, or however involved its probable detail appears, the student has become the true designer. It is the knowing how to attack a problem, to start definite work on it, to go ahead boldly, confident that the method applied will

produce results, that gives command of the design of machinery and wins engineering success.

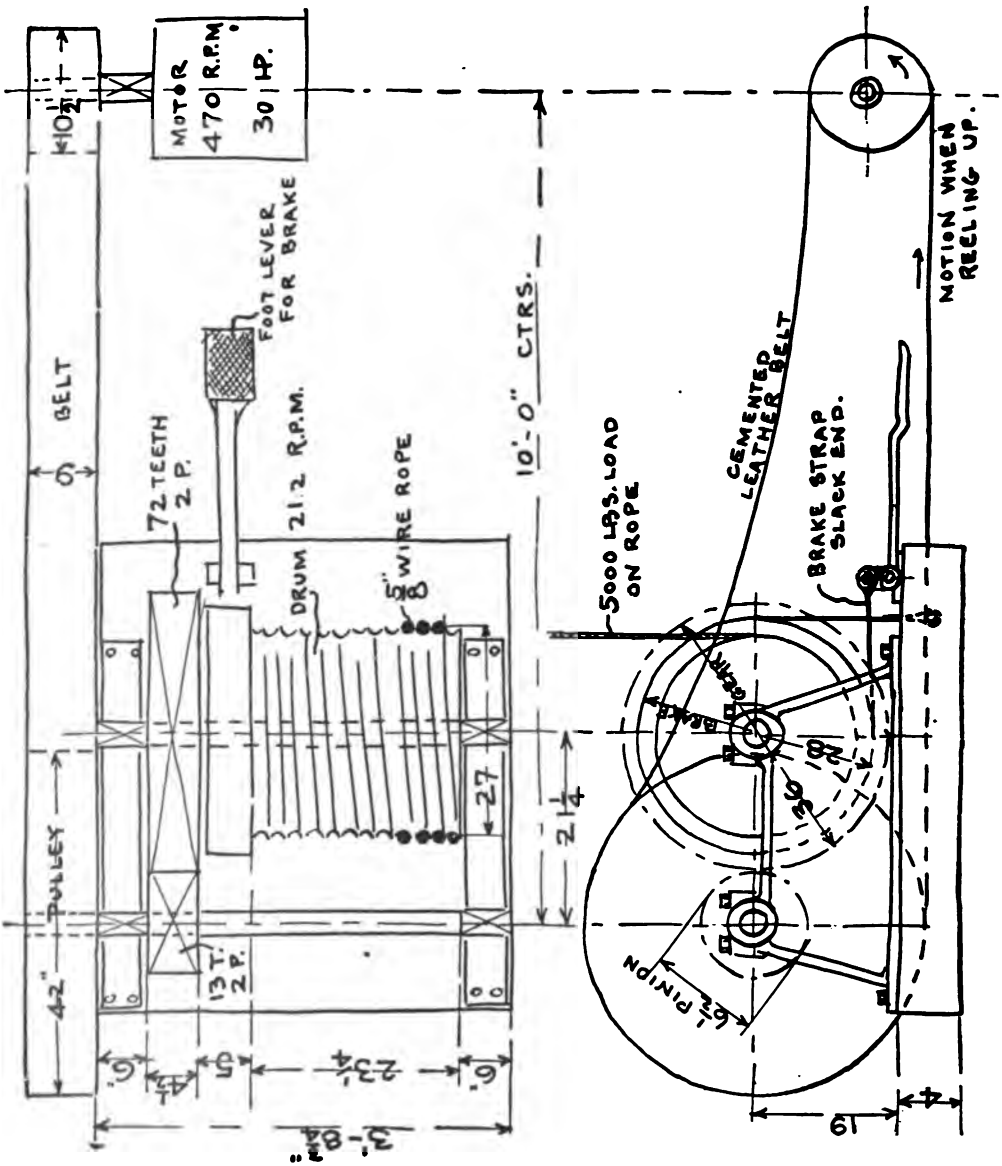


Fig. 1.

The special case which has been chosen to illustrate the application of the principles stated in the foregoing pages is ideal,

in that it does not represent any actual machine at present in operation. Probably builders of hoisting machinery have devices which would improve the machine as shown. In detail, as well as arrangement, they could doubtless make criticism as manufacturers. The arrangement as shown is merely intended to bring out in simplest form the common elements of transmission machinery as parts of some definite machine, instead of as isolated details. The design is one entirely possible, practical, and mechanical, but special attention has been paid to simplicity in order to enable the student to follow the method closely, for the *method* is the chief thing for him to acquire.

The student is expected to refer constantly to Part II for a more formal and general discussion of the simple machine elements involved in the case considered. Part II is intended to be a simplified and condensed reference book, carried out in accordance with the method of machine design as specified in Part I. The student should not wait until he has completed the study of this part before taking up Part II, for the latter is intended for use with the former in the solution of the problems.

In the case of power transmission about to be studied, the running, conversational method employed assumes that the student is in possession of the matter in Part II on the subject considered. Thus, in the design of the pulley, reference to the subject of "Pulleys" in Part II is necessary to follow the train of calculation; in designing the gear, consult "Gears;" in calculating size of shafts, see "Shafts," etc., etc.

Problem. A machine is to be designed to be set on the floor of a building to drive a wire rope falling from the overhead sheaves of an elevator or hoist. Without regard to details of this overhead arrangement, for its design would be a separate problem, suppose that the data for the rope are as follows:

Load on rope......................	5,000 pounds.
Speed of rope.....................	150 feet per minute.
Length of rope to be reeled in......	200 feet.

We shall further assume that the driving power is to be an electric motor belted to the machine, that the required speed reduction can be satisfactorily obtained by a single pair of pulleys and one pair of gears, and that a plain band brake is to be applied to the drum.

With this data we shall proceed to work out the detail design of the machine.

Preliminary Sketch. The first thing to do is to sketch roughly the proposed arrangement of the machine.

This might appear like Fig. 1 except that it would have no dimensions in addition to the data given above. If the scheme seems suitable, the next step is to make such preliminary calculations as will give further data, exact or closely approximate sizes, to be put at once on the sketch, to outline the future design.

Rope and Drum. Referring to tables of strength of wire rope (Kent's Pocket Book gives the manufacturers' list), we find that a $\frac{5}{8}$-inch cast-steel rope will carry 5,000 pounds safely, and that the proper size of drum to avoid excessive bending of the rope around it is 27 inches diameter.

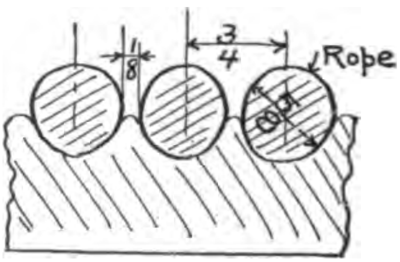


Fig. 2.

Allowing $\frac{1}{8}$ inch between the coils as the rope winds on the drum, the pitch of coil will be $\frac{3}{4}$ inch as shown in sketch, Fig. 2. The length of one complete coil is, practically, $\frac{27\times 3.1416}{12}$ $=7.07$ feet. To provide for 200 feet will require $\frac{200}{7.07}=28+$coils. To be safe, let us provide for 30 coils, for which a length of drum $(30\times\frac{3}{4})+\frac{3}{4}=23\frac{1}{4}$ inches is required.

The space for brake strap may be assumed at 5 inches, and the thickness to provide necessary strength determined later in the design. The frictional surface of the strap may be of basswood blocks, say $1\frac{1}{4}$ inches thick, screwed to the metal band. The diameter of brake surface may be 28 inches.

Driving Gears. The size of drum gear evidently depends upon the method of fastening to the drum, and, other things being equal, should be kept as small as possible. One way would be to key the gear on the outside of the drum, another to bolt the gear to the end of the drum. The latter has the advantage that a standard gear pattern can be used with the slight change of

addition of bolt flange on the arms. This makes a simple, direct, and strong drive, the bolts being in shear.

Sketching this arrangement as the preferred one (Fig. 2A), it is evident that the diameter of the gear should be at least as large as the drum in order to keep the tooth load down to a reasonable figure. On the other hand, if made too large, it spreads out the machine and destroys its compactness. As a diameter of 36 inches is not excessive, let us assume this, and see if a desirable proportion of gear tooth can be found to carry the load.

For a pitch diameter of 36 inches there will be a theoretical load of $\frac{5{,}000 \times 27}{36} = 3{,}750$ pounds at the pitch line. But the load

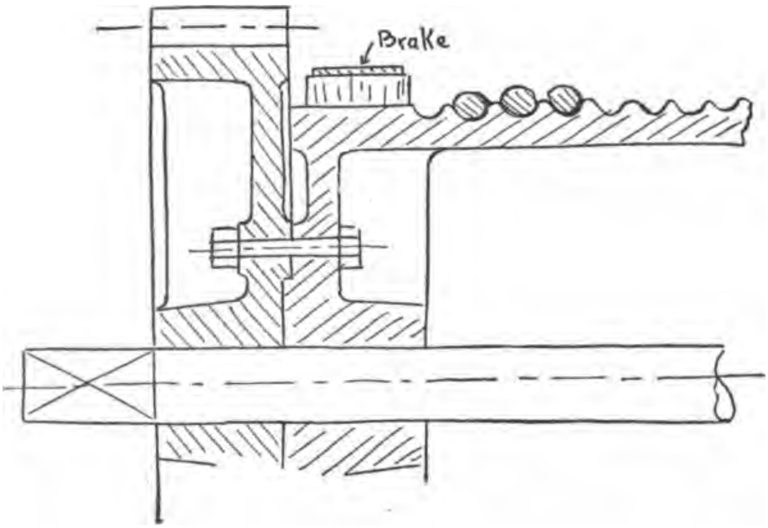


Fig. 2A.

on the tooth must not only impart a pull of 5,000 pounds to the rope, but must overcome friction between the gear teeth in action, also between the drum shaft and its bearings. Assuming the efficiency between the rope and tooth load to be 95 per cent, the net load, therefore, which the tooth must take is $\frac{3{,}750}{.95} = 3{,}947$, say 4,000 pounds.

Assuming involute teeth, and applying the "Lewis" formula, (Part II, "Gears"):

$$W = s \times p \times f \times y$$

$W = 4{,}000$
$s = 6{,}000$
$y = .116$ (number of teeth assumed at 75)
$p =$ circular pitch
$f =$ face of gear

$$4{,}000 = 6{,}000 \times p \times f \times .116$$

$$p \times f = \frac{4{,}000}{6{,}000 \times .116} = 5.7 \text{ inches}$$

Let $f = 3p$ (a reasonable proportion for machine-cut teeth).

Then $3 \times p^2 = 5.7$

$p^2 = 1.9$

$p = \sqrt{1.9} = 1.378$ inches

The diametral pitch corresponding to this is

$$\frac{3.1416}{1.378} = 2.28$$

which is just between the regular standard pitches, 2 and $2\frac{1}{2}$, for which stock cutters are made. To be safe, let us take the coarser pitch, which is 2. The circular pitch corresponding to this is $\frac{3.1416}{2} = 1.57$, and making the face about three times the circular pitch gives

$$3 \times 1.57 = 4.71, \text{ say } 4\tfrac{1}{2} \text{ inches.}$$

The number of teeth in the gear is then $36 \times 2 = 72$. Referring to the value assumed for the tooth factor in calculation above, it is seen that y was based on 75 as the number of teeth, which is near enough to 72 to avoid the necessity of further checking the result.

The pinion to mesh with this gear should be as small as possible in order to get a high-speed ratio between pinion shaft and drum, otherwise an excessive ratio will be required in the pulleys, making the large one of inconvenient size. Small pinions have the teeth badly undercut and therefore weak, 13 teeth being the lowest limit usually considered desirable, for that reason. Choosing that number, we have a pitch diameter of $\frac{13}{2} = 6.5$ in., which is probably ample to take the shaft and key, and still leave sufficient stock under the tooth for strength. If made of cast iron, however, the pinion teeth, on account of the low number, will be narrower at the root than those of the gear of 72 teeth. Yet it

was upon the basis of the latter that the pitch was chosen, for it will be remembered that the value of y in the formula was taken at .116. Hence the pinion will be weaker than the gear unless we make it of stronger material than cast iron, of which the large gear is supposed to be made. Steel lends itself very readily to this requirement; and in practice, pinions of less than 20 teeth are usually made of this material, hence we shall specify the pinion to be of steel.

Pulleys. The question now is whether or not we can get a suitable ratio in the pulleys without making the large one of inconvenient size, or giving the motor too slow speed for an economical proportion.

Suppose we limit ourselves to a diameter of 42 inches for the large pulley, and try a ratio of 4 to 1; this will give a diameter for the small pulley of $\frac{42}{4}=10\frac{1}{2}$ inches. We shall then have

Total ratio between drum and motor........	$\frac{72}{13}\times 4=\frac{288}{13}=22.2$
Rev. per min. of drum to give 150 f. p. m. of rope	$\frac{150}{7.07}=21.2$
Rev. per min. of motor......................	$22.2\times 21.2=470$
Horse-power of motor at 80 per cent efficiency	$\frac{150\ \times 5{,}000}{33{,}000\times\ .80}=30$

A 30 H. P. motor running 470 r. p. m. would be classed as a slow speed motor and would be a heavier machine and cost more than one of higher speed. It will be noticed, however, that the diameter of the small pulley is already quite reduced, and it is hardly desirable to decrease it still further. Neither can we increase the large pulley, as we have already set the limit at 42 inches. Hence, for our present problem we cannot improve matters much without increasing the size of the large gear, which is undesirable, or putting in another pair of gears, which is contrary to the conditions of the problem. As such a motor is perfectly reasonable, we shall assume it to be chosen for the purpose.

In commercial practice it would be well to pick out some standard make of motor of the required horse-power, note the speed as specified by the makers, and then, if possible, suit the ratio in the machine to this speed. It is always best to use standard machinery, if possible, both from the standpoint of first cost, as well

Width of belt. *May 20, 1903*

$$lg\ \frac{T_n}{T_0} = 2.729\,\mu(1-z)n \qquad \mu = .3$$

$$T_n - T_0 = P \qquad n = .5$$

$$z = \frac{1}{9660}\ \frac{wV^2}{t} \qquad t = 400 \text{ lbs.} \qquad w = .036 \text{ lbs.}$$

$$P = \frac{13541}{21} = 644.8 \text{ lbs.}$$

$$V = \frac{470 \times \overset{.2618}{\cancel{3.1416}} \times 10.5}{\cancel{12}} = 1292 \text{ (say 1300)}$$

$$\therefore z = \frac{1}{966\cancel{0}} \times \frac{\overset{.009}{\cancel{.036}} \times 169\cancel{0000}}{\cancel{400}} = \frac{15.210}{966} = .015$$

(.015 small, can be disregarded)

$$lg\ \frac{T_n}{T_0} = 2.729 \times 3 \times .5 = 0.409$$ for which the natural number is 2.56

$$\therefore \frac{T_n}{T_0} = 2.56 \qquad T_0 = \frac{T_n}{2.56}$$

$$T_n - T_0 = 645$$

$$T_n - \frac{T_n}{2.56} = \frac{1.56\,T_n}{2.56} = 645$$

$$\therefore T_n = \frac{645 \times 2.56}{1.56} = \underline{\underline{1059}}$$

$$T_0 = 1059 - 645 = \underline{\underline{414}}$$

$$T_n = b \times h \times t$$

b = belt width in.
h = " thickness " (say .3)
t = 400 (as above)

$$1059 = b \times .3 \times 400$$

$$b = \frac{1059}{.3 \times 400} = \underline{\underline{8.8}} \text{ (say 9" belt)}$$

(make pulley face $9\frac{1}{2}$")

Fig. 8

as ease of replacing worn parts. Machinery ordered special is expensive in first cost of designing, patterns, and tools, and extra spare parts for emergency orders are not often kept on hand.

Tabulation of Torsional Moments. For future reference, it is desirable at this point to tabulate the torsional moment, or torque, about each of the three shaft axes, assuming reasonable efficiencies for the various parts, as follows:

Efficiency between drum and gear tooth............95 per cent
Efficiency between drum and pinion shaft...........90 per cent
Efficiency between drum and motor shaft...........80 per cent

TABLE OF TORSIONAL MOMENTS.

Axis.	Inch Lbs. Torque at 100 Per Cent Efficiency.	Inch Lbs. Torque, Efficiency as Above.
Drum......	$5{,}000\times\frac{27}{2}$ ....$=67{,}500$	$\frac{67{,}500}{.95}=71{,}052$
Pinion......	$5{,}000\times\frac{27}{2}\times\frac{13}{72}$ ......$=12{,}187$	$\frac{12{,}187}{.90}=13{,}541$
Motor......	$5{,}000\times\frac{27}{2}\times\frac{13}{72}\times\frac{10.5}{42}=3{,}047$	$\frac{3{,}047}{.80}=3{,}809$

This means that the motor develops a torque of 3,809 inch-pounds delivering to pinion shaft 13,541 inch-pounds, and to drum 71,052 inch-pounds.

Width of Belt. The page of calculation for belt width is reproduced in Fig. 3.

The calculation as given is strictly scientific, based on the working strength of a cemented joint ($t=400$ lbs. per square inch). This is a favorable situation for the use of a cemented joint, because it is easy to provide means of adjusting the belt tension by placing the motor on a sliding base. Otherwise a laced joint could be used, requiring relacing when the belt slackens through its stretch in service. Under the assumption that a double *laced* belt is used, the empirical formula below is one often applied:

$$\text{H. P.}=\frac{w\times V}{540}=\frac{w\times 1{,}300}{540}=30$$

This gives $w=\frac{540\times 30}{1{,}300}=12.4$ inches (say 12 inches).

It should be remembered that this value is purely empirical; it applies to a *laced* joint, and could not be expected to check the

value of 9 inches obtained by the first computation for a cemented joint. It is fairly in proportion. For the quite definite service required of the belt in the present case, the width of 9 inches is doubtless sufficient, considering the cemented joint.

Length of Bearings. Considerable latitude in choice of length of bearings is permissible, especially in such slow-speed machinery. There is probably little danger from heating, and the question then becomes one of wear. It is better in such cases as the one in question, to choose boldly a length which seems to be reasonable and proceed with the design on that basis, even if the length be later found out of proportion to the shaft diameter, than to waste too much time in the preliminary calculation over the exact determination of this question. Probably in most cases of commercial practice the existence of patterns, or some other practical consideration, will decide the limits of length.

In the present instance it seems reasonable that a length of 6 inches would fill the requirement for the worst case, that of the drum shaft, and it is obvious that the bearings for the pinion shaft would naturally be of the same length on account of being cast on the same bracket, and faced at the same setting of the planer tool.

Height of Centers. The large pulley should naturally swing clear of the floor. This will require, say, a total height of 23 inches, out which we may take 4 inches for the base, leaving 19 inches as the height, center of bearing to base of bracket.

Data on Sketch. The data as found above should now be put on the sketch previously made; it will then have the appearance shown in Fig. 1.

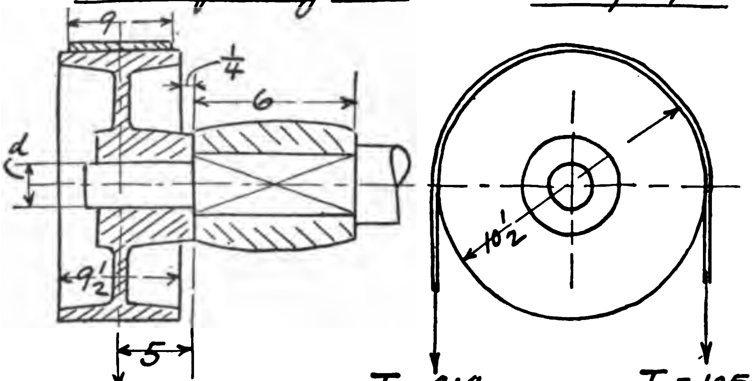
This sketch is now in form to control all the subsequent detail design, and it is expected that the figured dimensions as shown can be maintained. It is impossible to predict this with positiveness, however, as in the working out of the minor details certain changes may be found desirable, when, of course, they should be made.

The shaft sizes do not appear on this sketch, hence before proceeding further the several shaft diameters must be calculated.

Sizes of Shafts. The calculations of the shaft diameters are good instances of systematic engineering computations, hence they are reproduced in the exact form in which they were made. The student should learn a valuable lesson in making and recording

calculations by following these carefully. Note that each set of figures is independent, both in the statement of given data, as well as in the actual computation. Observe how easy it would be for the author of these figures or anyone else to check them even after

Small pulley bore. *May 21, 03*



$T_n + T_o = 1473$ $T_o = 414$ $T_n = 1059$

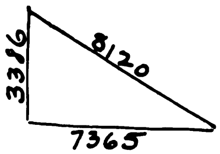
$B = 1473 \times 5 = 7365$

$T = 645 \times 5.25 = 3386$

$T_e = B + \sqrt{B^2 + T^2}$

$= 7365 + \sqrt{7365^2 + 3386^2}$

$= 7365 + 8120 = 15485$



$T_e = \frac{sI}{c} = \frac{sd^3}{5.1}$ let $s = 8000$

$15485 = \frac{8000\,d^3}{5.1}$

$d^3 = \frac{15485 \times 5.1}{8000} = 9.872$

$d = \sqrt[3]{9.872} = 2.14$ say $2\frac{1}{8}$

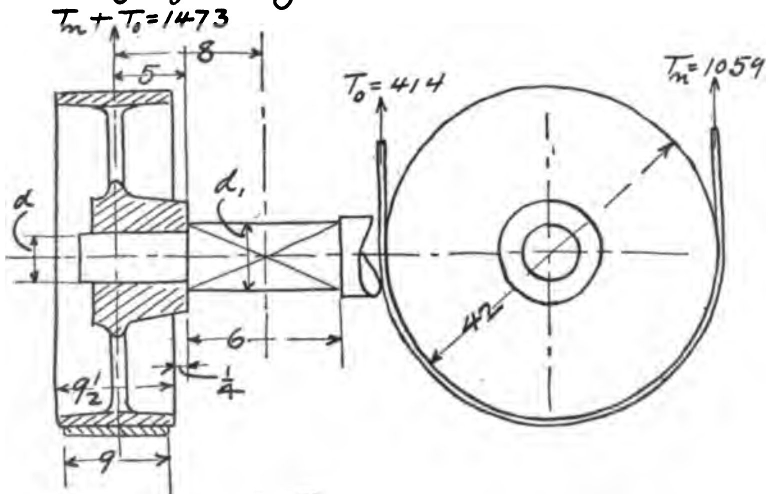
Fig. 4.

a long lapse of time. If the machine should unexpectedly fail in service the figures are always available to prove or disprove theoretical weakness. The right triangles merely indicate that the value of $\sqrt{B^2+T^2}$ was found by the graphical method suggested in

Part II, "Shafts," the figures being put on the triangle as a simple and direct way of recording both process and result.

Attention is especially called to the fact that in the pinion shaft the size is changed for each piece upon the shaft. This is

Large pulley bore   May 21 '03



$$B = 1473 \times 5 = 7365$$
$$T = 644.8 \times 21 = 13541$$
$$T_e = B + \sqrt{B^2 + T^2}$$
$$= 7365 + \sqrt{7365^2 + 13541^2}$$
$$= 7365 + 15425 = 22790$$
$$T_e = \frac{SI}{c} = \frac{Sd^3}{5.1} \quad \text{let } S = 8000$$
$$22790 = \frac{8000\, d^3}{5.1}$$
$$d^3 = \frac{22790 \times 5.1}{8000} = 14.529$$
$$d = \sqrt[3]{14.529} = 2.44 \quad \text{say } \underline{2\tfrac{7}{16}}$$

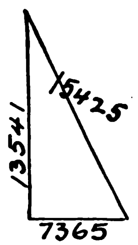


Fig. 5.

done partly because it is desired to show the student that the shaft at each of these points should be theoretically of different size. It is also done because as a practical feature of construction it is a good plan to change the size when the fit changes, partly for reasons of production in the shop, partly for ease in slipping pieces

freely endwise on the shaft until they reach their proper fit and location in the assembling of the machine.

This should not be taken as an absolute requirement in any sense. A straight shaft would be satisfactory in the present case; but the shouldered shaft is a little better construction, in a mechanical sense, and does not cost much more. Hence it is used. For the drum the straight shaft seems to answer the requirement well enough.

Bearing next to large pulley. May 21-03

see Fig. 5 and Fig. 7

$B = 1473 \times 8 = 11784$ (due to belt pull).

$B = 3458 \times 3 = 10374$ (due to load on pinion tooth)

Taking the greater value

$B = 11784$

$T = 644.8 \times 21 = 13541$

$T_e = B + \sqrt{B^2 + T^2}$

$= 11784 + \sqrt{11784^2 + 13541^2}$

$= 11784 + 17950 = 29734$

$T_e = \frac{SI}{c} = \frac{Sd_1^3}{5.1}$ let $S = 8000$

$29734 = \frac{8000\,d_1^3}{5.1}$

$d_1^3 = \frac{29734 \times 5.1}{8000} = 18.955$

$d_1 = \sqrt[3]{18.955} = 2.66$ say $2\frac{11}{16}$

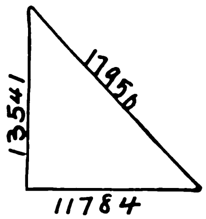


Fig. 6.

Small Pulley Bore. Fig 4.

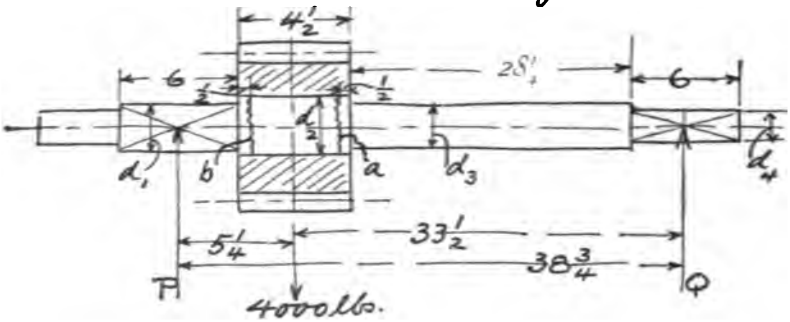
Large Pulley Bore. Fig. 5.

Bearing Next to Large Pulley. Fig. 6.

The diameter, $2\frac{11}{16}$, as calculated, is based on the supposition that the greatest bending moment is caused by the belt pull on the overhanging pulley, that is, by the forces existing at the left-hand side of the center of the bearing.

But the pinion tooth load produces a heavy bending on the shaft in the bearing, the shaft in this case acting as a beam sup-

Pinion Bore May 21 '03



$P \times 38.75 = 4000 \times 33.5$ (moments about Q)

$$P = \frac{4000 \times 33.5}{38.75} = 3458$$

$$Q = 4000 - 3458 = 542$$

$B = 542 \times 31.75 = 17208$ (due to Q)

$B = 3458 \times 3.5 = 12103$ (due to P)

Taking the greater value

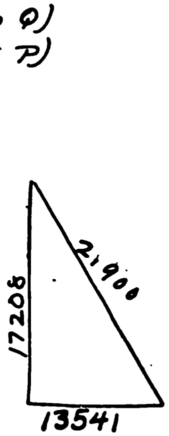
$B = 17208$

$T = 13541$ (as before)

$$T_e = B + \sqrt{B^2 + T^2}$$

$$= 17208 + \sqrt{17208^2 + 13541^2}$$

$$= 17208 + 21900 = 39108$$



$T_e = \frac{SI}{c} = \frac{S d_2^3}{5.1}$ let $S = 8000$

$$39108 = \frac{8000 d_2^3}{5.1}$$

$$d_2^3 = \frac{39108 \times 5.1}{8000} = 24.928$$

$d_2 = \sqrt[3]{24.928} = 2.92$ say $2\frac{15}{16}$

Fig. 7.

ported at the two bearings and having the tooth load applied as shown. If this latter effect be greater than the former, that is, if

the bending moment produced by the pinion tooth load be greater than the bending moment produced by the belt pull, then the diameter must be increased to satisfy the latter case. As is seen by the second calculation of Fig. 6, this is not the case, and the diameter stands at $2\frac{1}{16}$ as made.

Pinion Bore. Fig. 7. The pinion being a driving fit upon the shaft, reinforces the shaft to such an extent that it is hardly possible for the shaft to break off very far inside the face of the pinion; but it is quite possible that the metal of the pinion may give enough, or be a little free at the ends of the hole, so that the shaft may be broken off, say $\frac{1}{2}$ inch inside the face. In this case, it may fail from the moment of the force at the left-hand bearing or of that at the right. It may fail then at (a) or (b), depending on which section has the greater bending moment. Trying both, it is seen by the calculation that the right-hand moment is the controlling one, and it, therefore, is used.

Shaft Outside of Pinion. Fig. 8. As there is no power transmitted through this portion of the shaft, there is no torsional moment in it, and the bending moment remains practically the same as inside the pinion.

The size figures about $2\frac{1}{16}$, but since there is no use in turning off material just to reduce the size to this, it is well to make it $2\frac{7}{8}$, or just smaller than the fit in the pinion.

Pinion Shaft Outer Bearing. Fig. 8. This diameter, of course, figures small, as there is no torsion in it, and the bending moment is not heavy. The practical question comes in, however, whether it is advisable to make the outer bracket different from the inner one just on account of this bearing. The commercial answer to this would probably be "No," hence the size as figured next to the pinion will be maintained ($2\frac{1}{16}$).

Drum Shaft. Fig. 9. In this case, as previously inferred, the simplest thing to do is to use a piece of straight cold-rolled steel, and make both bearings alike, the size being determined according to the worst case of loading which can occur as the rope travels from end to end of the drum. This case is evidently when the rope is at the end of its travel close to the brake, for at that time both the load on the rope and the load on the pinion tooth which is driving it are exerted upward, and produce the

greatest reaction at the bearing next to the gear. The analysis of the forces for this condition is shown in Fig. 9.

Other conditions of loading would be when the brake is on and the tooth load relieved, but then the resultant of the brake strap tensions would be diagonally downward and would reduce

Shaft outside of pinion. May 22-'03

See sketch of Fig. 7

$B = 542 \times 31.25 = 16937 \qquad T = 0$

$B = \frac{SI}{c} = \frac{Sd_3^3}{10.2} \qquad \text{let } S = 8000$

$16937 = \frac{8000\, d_3^3}{10.2}$

$d_3^3 = \frac{16937 \times 10.2}{8000} = 21.594$

$d_3 = \sqrt[3]{21.594} = 2.78 \quad \text{say } \underline{\underline{2\frac{13}{16}}}$

Pinion shaft outer bearing. May 22-'03

See sketch of Fig. 7

$B = 542 \times 3 = 1626 \qquad T = 0$

$B = \frac{SI}{c} = \frac{Sd_4^3}{10.2} \qquad \text{let } S = 8000$

$1626 = \frac{8000\, d_4^3}{10.2}$

$d_4^3 = \frac{1626 \times 10.2}{8000} = 2.073$

$d_4 = \sqrt[3]{2.073} = 1.27 \quad \text{say } \underline{\underline{1\frac{5}{16}}}$

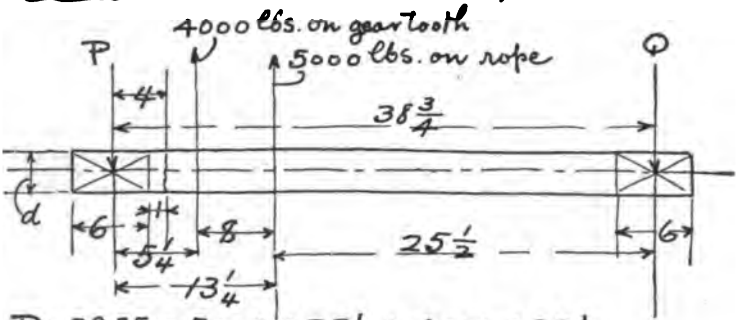
Fig. 8.

rather than add to the rope load. Again, when the rope is at the end of the drum farthest from the gear, the load on it and the load on the pinion tooth are both exerted upward as before, but the reaction cannot be as great as in the case of Fig. 9, because the tooth load is still concentrated at the other end of the shaft and produces a relatively small reaction at the rope end

Preliminary Layout. Fig. 10. Proceeding now with the layout to scale, the detail of the parts may be worked out as completely as the scale of the drawing will permit. The work on this drawing may be of an unfinished, sketchy nature, but the measurements must be exact as far as they go, for this drawing is to serve as the reference sheet, from which all future detail is to be worked up.

In this layout may be worked out the sizes of the arms and hubs of pulleys and gears, the proportions of the drum and brake

Drum shaft May 22'03



$$P \times 38.75 = 5000 \times 25\tfrac{1}{2} + 4000 \times 33\tfrac{1}{2}$$

$$P = \frac{261500}{38.75} = 6748$$

$$B = 6748 \times 4 = 26992$$

$$B = \frac{SI}{c} = \frac{Sd^3}{10.2} \qquad \text{let } S = 8000$$

$$26992 = \frac{8000\,d^3}{10.2}$$

$$d^3 = \frac{26992 \times 10.2}{8000} = 34.415$$

$$d = \sqrt[3]{34.415} = 3.25 \quad \text{say } 3\tfrac{1}{4}$$

Fig. 9.

strap, and the general dimensions of the side brackets and the base. When the detail becomes too fine to work out to advantage on this drawing it may be worked out full size by a separate sketch, or left to be finished when it is regularly detailed. The preliminary layout, it should be remembered, is a service sheet only, a means of carrying along the design, and not intended for

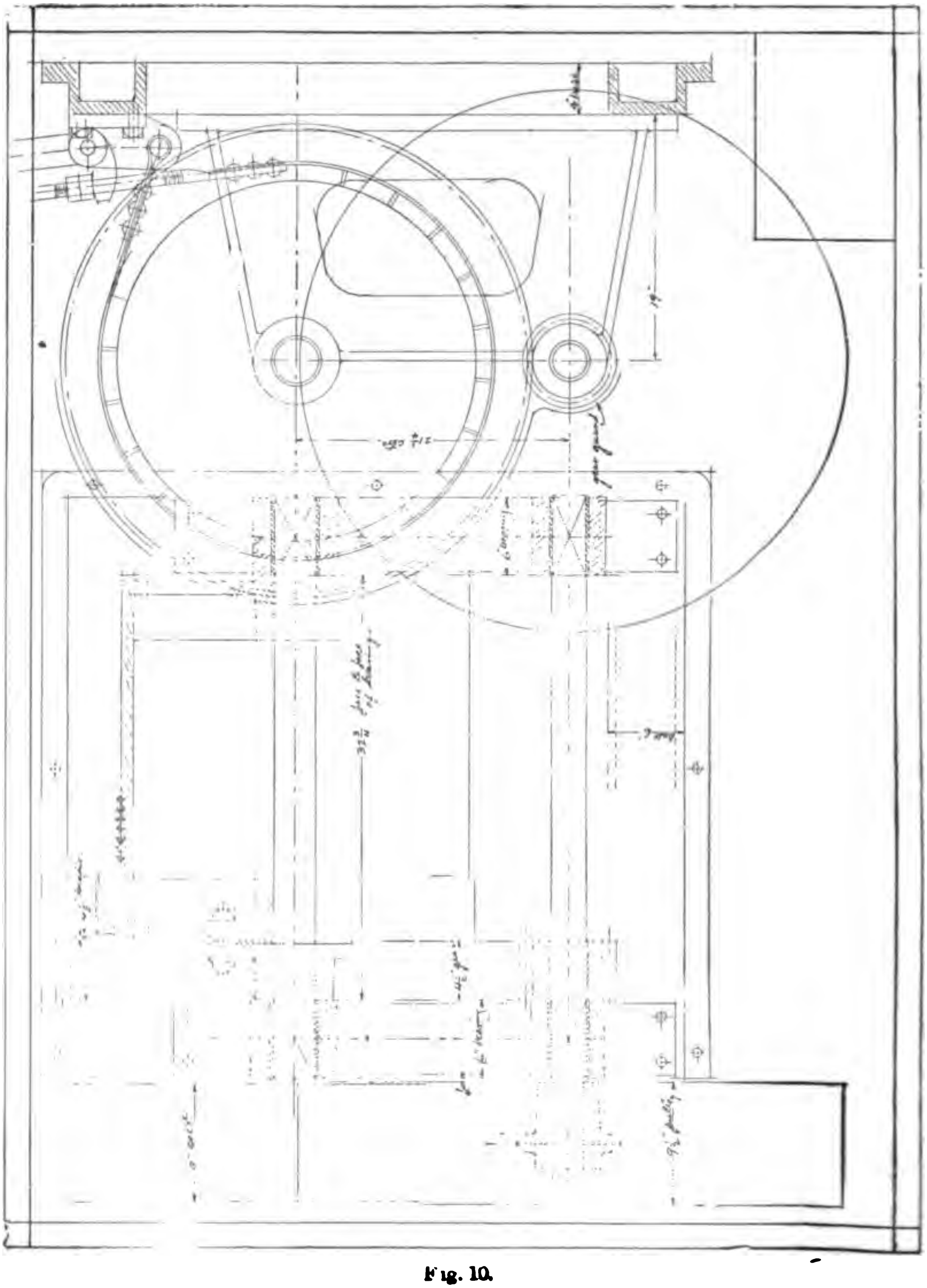


Fig. 10.

a finished drawing. The moment that the free use of the layout is impaired by trying to make too much of a drawing of it, its value is largely lost. A designer must have some place to try out his schemes and devices, and the layout drawing is the place to do it. This drawing may be recurred to at intervals in the progress of the design, details being filled in as they are worked out, as they may control the design of adjacent parts.

As the discussion of the design of each of the members involved in the present problem can be better taken up in connection with the detail drawing of each, it will be given there, rather than in connection with the layout, although many of the proportions thus discussed could be worked out directly from the latter.

Pulleys. Fig. 11. The analysis of the forces in the belt gives, according to the calculation of Fig. 3, a tension in the tight side of 1,059 pounds, and in the slack side 414 pounds. The difference of these, or 1,059—414=645 pounds, is transmitted to the pulley and produces the torque in the shaft. Of course in the small pulley the torque is transmitted from the motor through the pulley to the belt, but both cases are the same as far as the loading of the pulleys is concerned.

The only other force theoretically acting is the centrifugal force due to the speed of the pulley. This produces tension in the rim and arms, but for the low value of 1,300 feet per minute peripheral velocity in this case may be disregarded.

Considering the arms as beams loaded at the ends, and that one-half the whole number of arms take the load, and for convenience, figuring the size of the arms at the center of the pulley gives the following calculation for the large pulley:

$$\frac{645}{3}\times 21=\frac{S\times I}{c}=.0393\times 2{,}500\times h^3$$

Let $S=2{,}500$

" h=breadth of oval

" $.4h$=thickness of oval

$$h^3=\frac{4{,}515}{98.25}=46$$

$$h=\sqrt[3]{46}=3.6 \text{ (say 3.5)}$$

$$.4h=.4\times 3.5=1.4 \text{ (say } 1\tfrac{7}{16})$$

This is about all the theoretical figuring necessary on this pulley. The rim is made as thin as experience judges it capable of being cast; the arms are tapered to suit the eye, thus giving ample fastening to the rim to provide against shearing off the rim

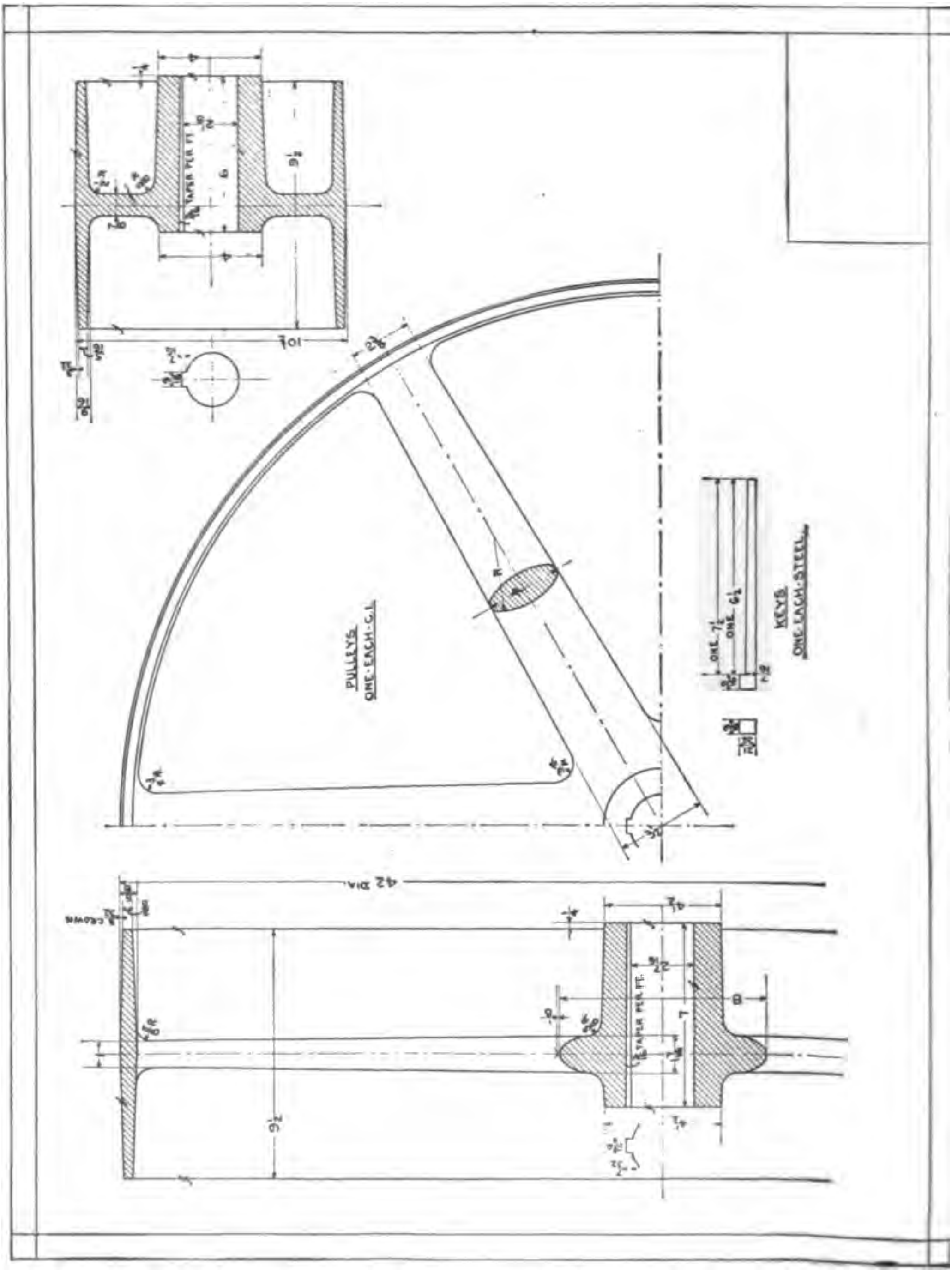

PULLEYS
ONE-EACH-C.I.
KEYS
ONE-EACH-STEEL
42 DIA.
CROWN
TAPER PER FT.

Fig. 11

from the arms; generous fillets join the arms to both rim and hub; and the hub is given thickness to carry the key, and length enough to prevent tendency to rock on the shaft. Uncertain strains due to unequal cooling in the foundry mold may be set up in the arms and rim, but with careful pouring of the metal they should not be serious, and the low value chosen for the fibre stress allows considerable margin for strength.

The small pulley has the same forces to withstand as the large pulley, but on account of its small diameter there is not room enough for arms between the rim and the hub, hence it is made with a web. The web cannot be given any bending by the belt pull, the only tendency which exists in this case being a shearing where the web joins the hub. This shearing also exists throughout the web as well, but at other points farther from the center it is of less magnitude, and moreover, there is more area of metal to take it. The natural way to proportion the thickness of the web is to give it an intermediate thickness between that of the hub and rim, thus securing uniform cooling, and then figure the stress as a check. Making this value $\frac{7}{8}$ inch gives a shearing area of $\frac{7}{8}$ multiplied by the circumference of the hub, which is $3.1416 \times 4 = 12.56$. The shearing force at the hub is $\frac{645 \times 5.25}{2} = 1{,}693$ pounds. Equating the external force to the internal resistance

$$1{,}693 = \tfrac{7}{8} \times 12.56 \times S$$

$$S = \frac{1{,}693 \times 8}{7 \times 12.56} = 154 \text{ pounds per square inch (approx.).}$$

This is a very low figure, even for cast iron, hence the web is amply strong. The rim and hub are proportioned as for the large pulley.

The keys are taken from the standard list. They may be checked for shear, crushing in the hub, and crushing in the shaft, but the hubs are so long that it is at once evident without figuring that the stress would run very low in both cases.

Gears. Fig. 12. The analysis of the forces acting on the gears has been given on page 28, 4,000 pounds being taken at the pitch line. Using this same value, and choosing a T-shaped arm as a good form for a heavily loaded gear like the present one, let us consider that the rim is stiff enough to distribute the load

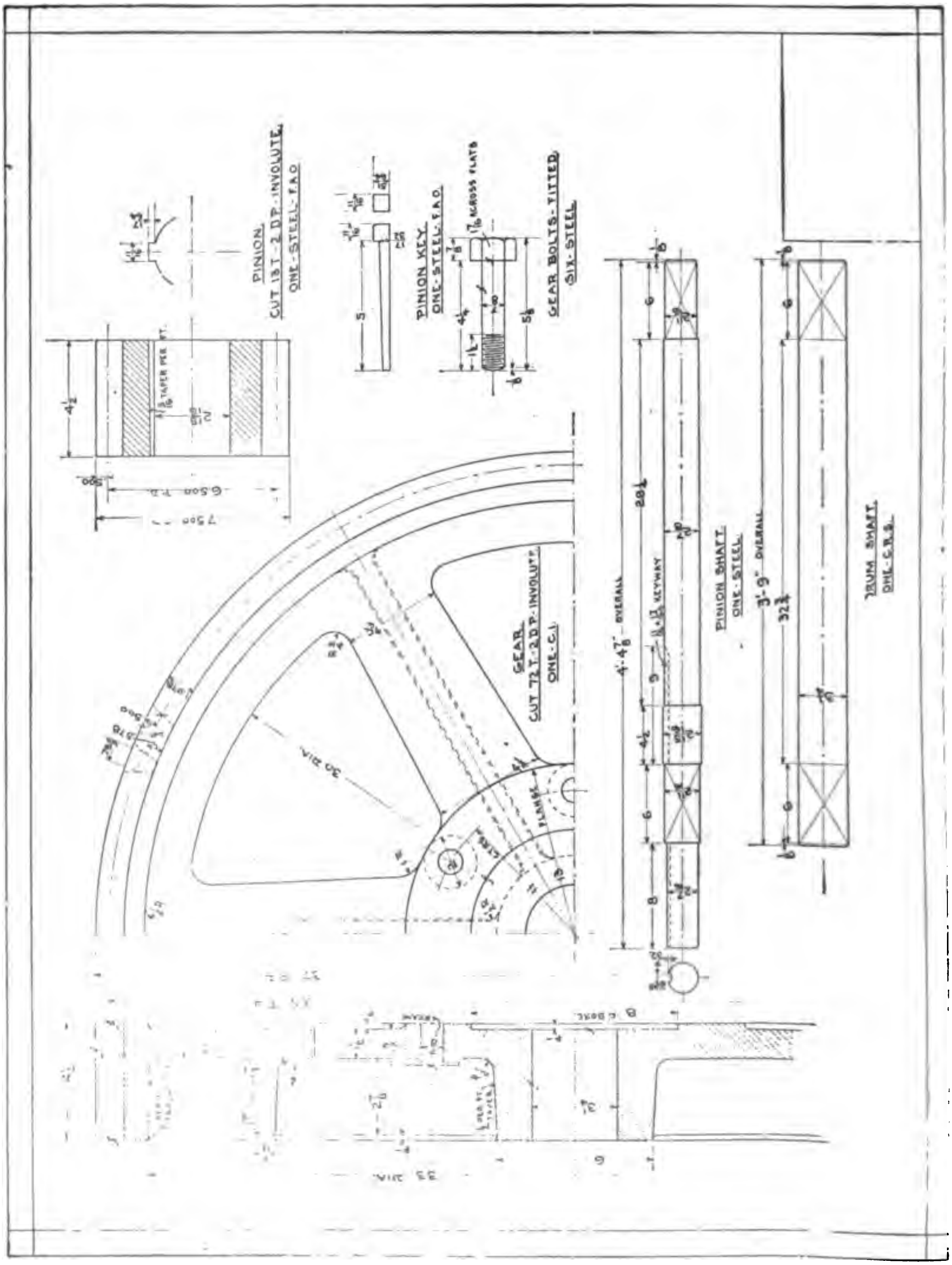

PINION
CUT 13T-2 D.P.-INVOLUTE
ONE-STEEL-F.A.O.
PINION KEY
ONE-STEEL-F.A.O.
GEAR BOLTS-FITTED
SIX-STEEL
GEAR
ONE-C.I.
PINION SHAFT
ONE-STEEL
DRUM SHAFT

Fig. 12.

equally between all the arms, and that each acts as a beam loaded at the end with its proportion of the tooth load. Before we can determine the length of these arms, however, we must fix upon the size of the flange which is to carry the driving bolts. This is taken at 13 inches. It could be smaller if desired, but drawing the bolts in toward the center increases the load on them, and 13 inches seems reasonable until it is proved otherwise. This makes the maximum moment which can come on an arm $\frac{4,000 \times 11.5}{6} = 7,666$ inch-pounds.

Now it is evident that the base of the T arm section, which lies in the plane of rotation, is most effective for driving, and that the center leg of the T does not add much to the driving capacity of the arm, although it increases the lateral stiffness of the arm, as well as providing in casting a free flow of metal between the rim and the hub. Hence the simplest way of treating the section of the arm for strength is to consider the base of the T only, of rectangular section, breadth b, and depth h, for which the internal moment of resistance is $\frac{S \times b \times h^2}{6}$.

Also, it is simplest to assume one dimension, say the breadth, and the allowable fibre stress, and figure for the depth. Taking the breadth at $1\frac{1}{8}$ inches, which looks about right, and the fibre stress at 2,500, and equating the external moment to the internal, we have

$$7,666 = \frac{2,500 \times 1.125 \times h^2}{6}$$

$$h^2 = \frac{6 \times 7,666}{2,500 \times 1.125} = 16.4$$

$$h = \sqrt{16.4} = 4.05 \text{ (say } 4\tfrac{1}{4}\text{)}$$

Drawing in this size, and tapering the arm to the rim as in the case of the pulleys, making the depth of the rim according to the suggested proportions given in Part II, "Gears," giving the center leg of the T a thickness of $\frac{7}{8}$ inch tapering to 1 inch, and heavily filleting the arms to the rim and center flange, we have a fairly well proportioned gear.

The next thing to determine is the size of the driving bolts. The circle upon which their centers lie may be 11 inches in diam-

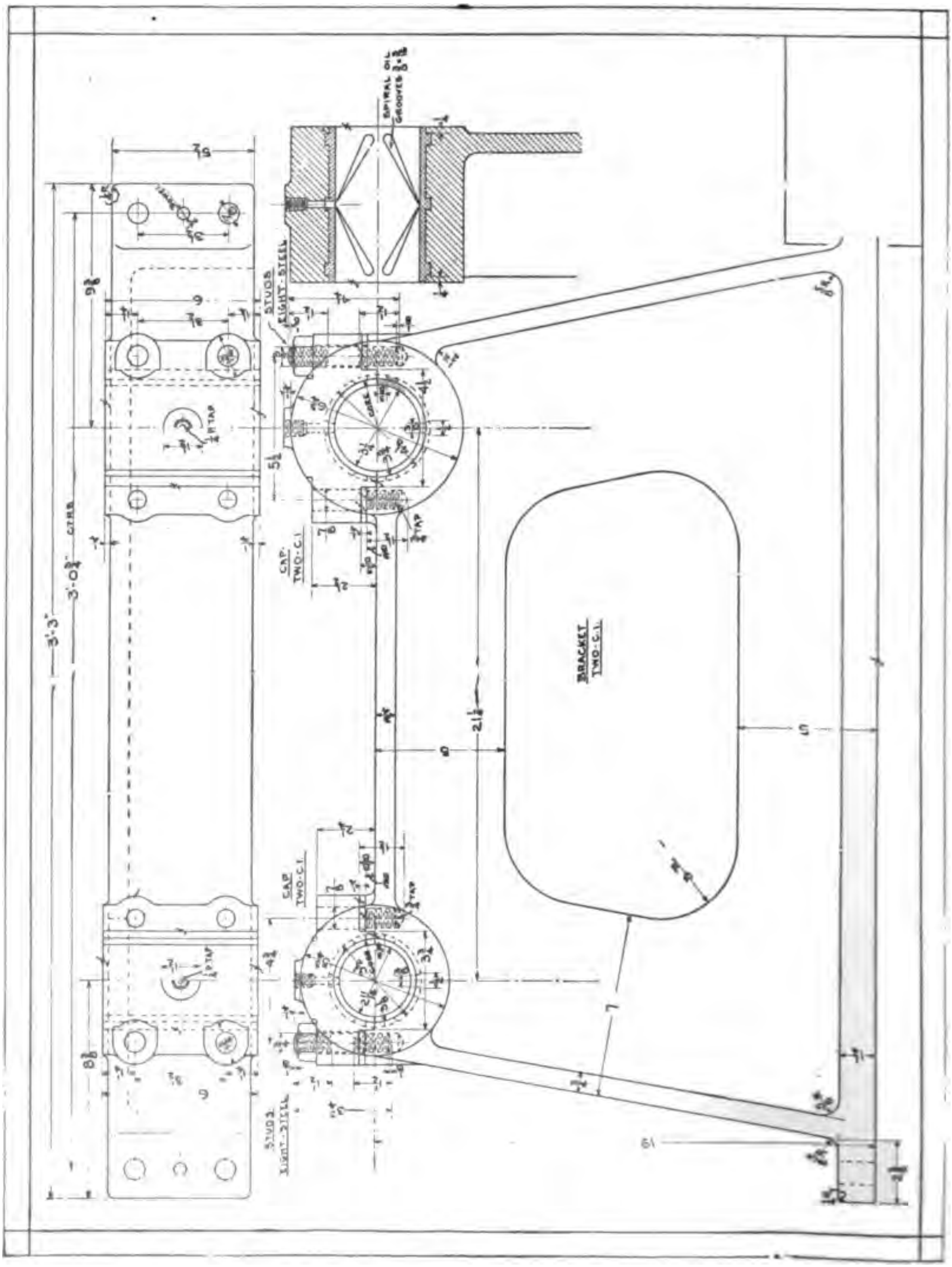

BRACKET
TWO-C.I.
CAP
TWO-C.I.
STUDS
EIGHT-STEEL
SPIRAL OIL
GROOVES

Fig. 13.

eter, and there will naturally be six bolts, one between each arm. These bolts are in pure shear, and the material of which they are to be made ought to be good for at least 8,000 pounds per square inch fibre stress. The force acting at the circumference of an 11-inch circle would be $\frac{4,000 \times 18}{5.5}=13,091$ pounds.

Equating the load on each bolt to the resisting shear gives

$$\frac{13,091}{6}=8,000 \times A=\frac{8,000 \times 3.1416 \times d^2}{4}$$

Let A = area resisting shear.
Let d = dia. of bolt.
Then $A=\frac{\pi d^2}{4}$

$$d^2=\frac{4 \times 13,091}{6 \times 8,000 \times 3.1416}=.35$$

$d=\sqrt{.35}$ (say .6) $\frac{5}{8}$-inch bolts would do.

But $\frac{5}{8}$-inch bolts are pretty small to use in connection with such heavy machinery. They look out of proportion to the adjacent parts. Hence $\frac{7}{8}$-inch bolts have been substituted as being better suited to the place in spite of the fact that theoretically they are larger than necessary. The extra cost is a small matter. These bolts may crush in the flange as well as shear off, but as there is an area of $\frac{7}{8} \times 1\frac{5}{8}=1.422$ square inches to take $\frac{13,091}{6}=2,182$ pounds, the pressure per square inch of projected area is only $\frac{2,182}{1.422}=1,534$ pounds, which is very low.

This gear needs no key to the shaft because all the power comes down the arms and passes off to the drum through the bolts, thus putting no torsional stress in the shaft. The face of the flange is counterbored so as to center the gear upon the drum, without relying upon the fit of the gear upon the shaft to do this.

The pinion is solid and needs no discussion for its design.

Brackets and Caps. Fig. 13. As the size of the drum shaft was determined by considering the rope wound close up to the brake, thus giving in combination with the load on the gear tooth the maximum reaction at the bearing as 6,718 pounds, the cap and bolts should be designed to carry the same load.

For a bearing but 6 inches long, two bolts are sufficient under ordinary conditions and might perhaps do for this case. The load is pretty heavy, however, and it is deemed wise to provide four bolts, thus securing extra rigidity, and permitting the use of bolts

of comparatively small size. If the load were distributed equally over all the bolts each would take one-fourth of the whole load, but it is not usually safe to figure them on this basis, because it is difficult to guarantee that each bolt will receive its exact share of stress. Assuming that the two bolts on one side take $\frac{2}{3}$ the whole load instead of $\frac{1}{2}$, which provides for this uncertain extra stress, each bolt must take care of $\frac{1}{3}$ of 6,748, or 2,249, pounds. Allowing 8,000 pounds per square inch fibre stress calls for an area at the root of the thread of $\frac{2,249}{8,000} = .281$ square inch. Consulting a table of bolts we find that the next standard size of bolt greater than this is $\frac{3}{4}$, which gives an area of .302 square inch.

Choosing this size as satisfactory, the bolts should be located as close to the shaft as will permit the hole to be drilled and tapped without breaking out. A center distance of $5\frac{1}{2}$ inches accomplishes this result. The distance between centers in the other direction is somewhat arbitrary, although the theoretical distance between the bolt and the end of the bearing to give equal bending moment at the center of the cap and at the line of the bolts is about $\frac{5}{24}$ of the length, or $\frac{5}{24}$ of $6 = 1\frac{1}{4}$ inches. This proportion answers well for the present case, although for long caps it brings the bolts too far in to look well.

The thickness of the cap may be determined by assuming it to be a beam supported at the bolts and loaded at the middle. This is not strictly true, for the load is distributed over at least a portion of the shaft diameter; moreover, the bolts to some extent make the beam fixed at the ends. It being impossible to determine the exact nature of the loading, we may take it as stated, supported at the ends and loaded in the middle, and allow a higher fibre stress than usual, say 3,500. The longitudinal section at the middle of the cap is rectangular, of breadth 6 inches, and depth unknown, say h. The equation of moments is

$$\frac{W \times l}{4} = \frac{S \times I}{c} = \frac{S \times b \times h^2}{6}$$

$$\frac{6,748 \times 5.5}{4} = \frac{3,500 \times 6 \times h^2}{6}$$

$$h^2 = \frac{6 \times 6,748 \times 5.5}{4 \times 3,500 \times 6} = 2.65$$

$$h = \sqrt{2.65} = 1.62 \text{ } (1\tfrac{1}{2} \text{ will probably answer})$$

For the other bearing next to the pinion, the load on the tooth acts downward, and the resultant pull of the belt is nearly horizontal, hence the cap and bolts must stand but little load, and calculation would give minute values. In a case like this it is well to make the size the same as for the larger bearing, unless the construction becomes very clumsy thereby. This saves changing drills and taps in making the holes, and preserves the symmetry of the bracket. The $\frac{3}{4}$-inch bolts are good proportion for the smaller bearing, hence that size will be maintained throughout.

The body of the bracket is conveniently made with the web at the side and horizontal ribs extending to the outside. The load due to the rope is carried directly down the side ribs and web into the bottom flanges and to the bolts. The analysis of the forces on these bolts is shown in Fig. 14. It is evident from the figure that the resultant belt pull tends to hold the bracket down, while the load on the rope tends to pull it up, the point about which it tends to rotate being the corner furthest from the drum. It is also evident that the bolts nearest this corner can have little effect on the holding down, because their leverage is so small about the corner, hence we shall assume that the pair of bolts at the right-hand end of the bracket takes all the load. The belt pull, being horizontal, tends to slide the bracket along the base, but this tendency is small, and at any rate is easily taken care of by the two dowel pins, which are thus put in shear.

The load on the bolts being 4,954 pounds, a heavy bending moment is thrown on the flange of the bracket, tending to break it off at the root of the fillet. The distance to the root of the fillet is $\frac{3}{4}$ inch; the section tending to break is rectangular, of breadth $5\frac{1}{2}$ inches, and unknown depth h. The equation of moments is

$$W \times l = \frac{S \times I}{c} = \frac{S \times b \times h^2}{6}$$

$$\frac{4{,}954 \times 3}{4} = \frac{2{,}500 \times 5.5 \times h^2}{6}$$

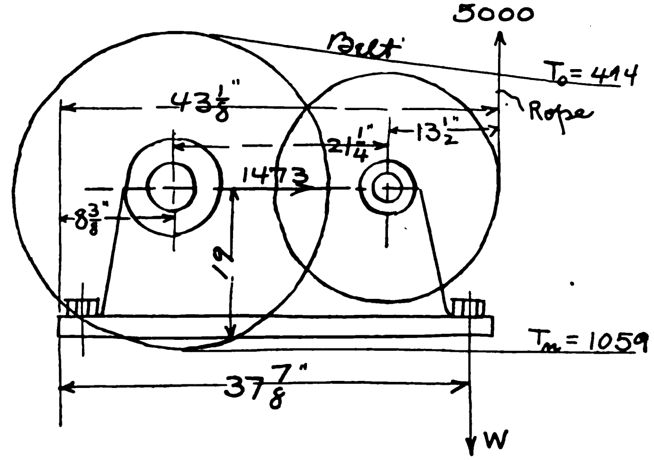
$$h^2 = \frac{6 \times 4{,}954 \times 3}{4 \times 2{,}500 \times 5.5} = 1.62$$

$$h = \sqrt{1.62} = 1.3 \text{ (say } 1\tfrac{1}{4}).$$

The thickness of the web and ribs of this bracket is hardly capable of calculation. The figure $\frac{3}{4}$ inch has been chosen in pro-

portion to the size of the large drum bearing, giving ample stiffness and rigidity, and permitting uniform flow and cooling of the metal in the mold. The opening in the center is made merely to save material, as in that part little stress would exist, the two sides

Bracket base bolts May 25-1903



$$5000 \times 43\tfrac{1}{8} = 37\tfrac{7}{8} \times W + 1473 \times 19$$

$$W = \frac{5000 \times 43.125 - 1473 \times 19}{37.875}$$

$W = 4954$ lbs on two bolts

$\frac{4954}{2} = 2477$ lbs per bolt.

$\frac{2477}{8000} = .31$ sq. in. area at 8000 lb. stress.

Area at root of thread for a $\frac{3}{4}$ bolt is .28

Hence $\frac{3}{4}$ bolts will answer.

Fig. 14.

carrying the load down to the base bolts, and the top serving as a tie between the bearings.

This bracket might be made with the web in the center of the bearings instead of at the side, in which case the expense of the

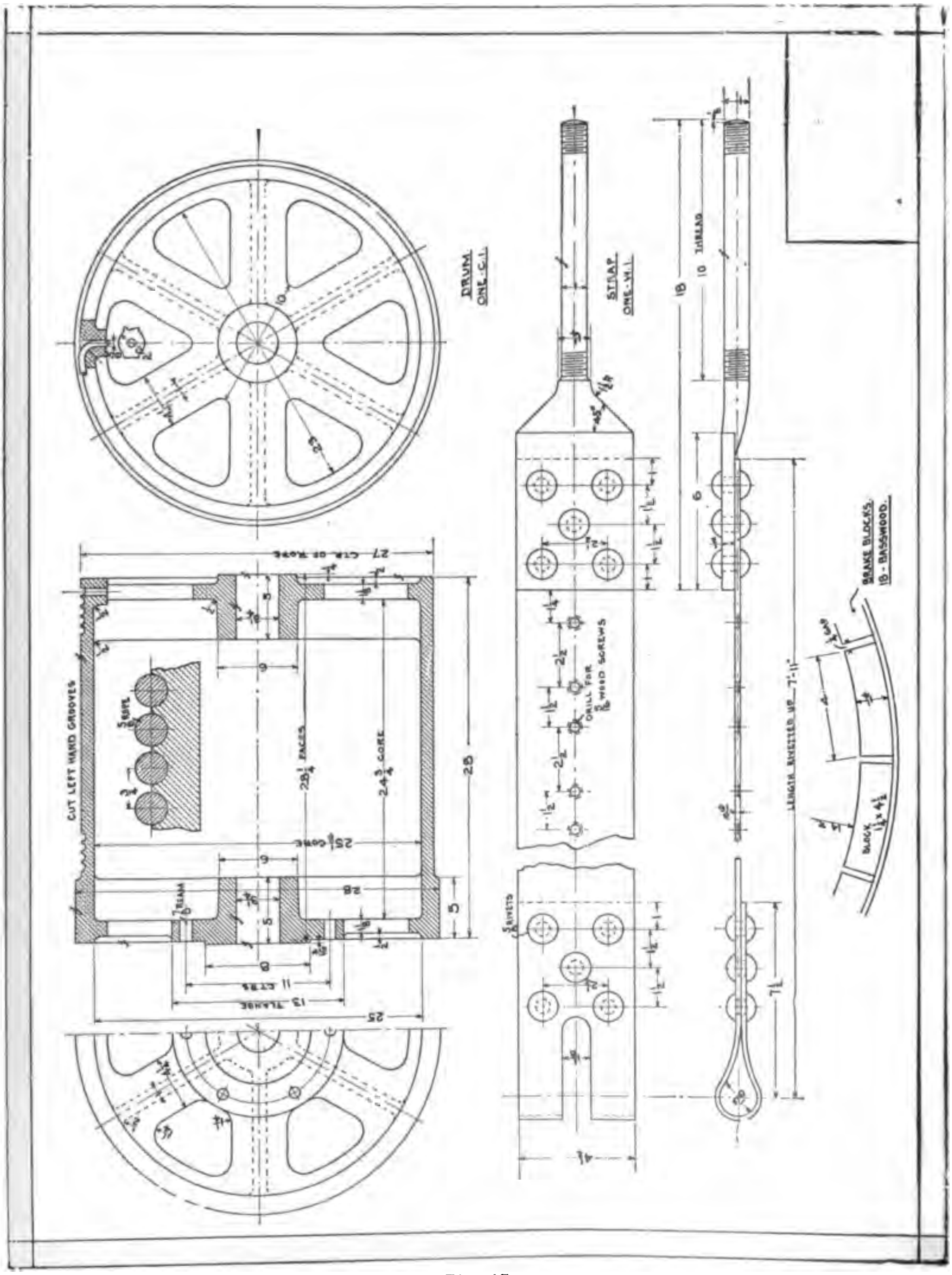
pattern would be slightly greater. It could also be made of closed box form, but would in that case probably weigh more than as shown.

Drum and Brake. Fig. 15. The analysis of the forces acting on the drum is simple, but its theoretical design is more complicated. It is evident that the drum acts as a beam of hollow circular cross section, and that its worst case of loading is when the rope is at or near the middle of the drum length. At the same time the metal of this circular cross section is in a state of torsion between the free end of the rope and the driving gear, due to the load on the gear tooth and the reaction of the rope. Also the wrapping of the rope around the drum tends to crush the metal of the section beneath it, the maximum effect of this action being near the free end of the rope where its tension has not been reduced by friction on the drum surface.

Now the "mechanics" to solve the problem of these three combined actions is rather complicated. It can be at least approximately solved, however, for it satisfies fairly well the case of combined compression and shear. But on a further study of this particular case, it is seen at once that the diameter of the drum is relatively large with respect to its length, which means that the thickness of the metal may be very small and yet give a large resisting area, or value of "I," both in direct bending as well as torsion; also it is so short that the external bending moment will be small. The practical condition now comes in, that the drum can be safely cast only when the thickness of the metal is at a minimum limit, for the core may be out of round, not set centrally, or by some other variation produce thin spots or even develop holes reaching out into the rope groove, discovered only when the latter is turned in the lathe.

Hence it seems reasonable and safe in this case to make the thickness of the drum depend simply upon the crushing caused by the wrapping of the rope around it, and we shall take the coil nearest the free end of the rope, assuming that it carries the full load of 5,000 pounds throughout one complete wrap around the drum.

The area resisting the crushing action may be considered to be that of the cross section of a ring, of width equal to the pitch



DRUM
ONE-C.I.
STRAP
ONE-W.I.
CUT LEFT HAND GROOVES
BRAKE BLOCKS
18 - BASSWOOD
10 THREAD

Fig. 15.

of the groove. Assuming that $\frac{5}{8}$ inch is the least thickness which can be safely allowed under the groove for casting purposes, let us figure the crushing fibre stress to see if this is sufficiently strong. Disregarding the small amount of metal existing above the bottom of the groove, this gives the area to resist the crushing $\frac{5}{8} \times \frac{3}{4} = \frac{15}{32}$, or .47 inch. Since there are two of these sections and the rope acts on both sides, the equation of forces is:

$$5{,}000 \times 2 = S \times .47 \times 2$$

$$S = \frac{5{,}000 \times 2}{.47 \times 2} = 10620 \text{ pounds per square inch.}$$

This, for cast iron, in pure crushing, allows plenty of margin for the extra bending and torsional stress, which for such a considerable thickness would be slight.

The above case indicates a method of reasoning much used in designing machinery, which while following out the specified routine of thought as previously given in these pages, stops short of elaborate and minute theoretical calculation when such is obviously unnecessary. If a drum of great length were to be designed, and of small diameter, the same method of reasoning would deduce the fact that the design should be based on the bending and the torsional moments, the thickness in such a case being so great to withstand these that the intensity of the crushing due to wrap of the rope becomes of inappreciable value.

The remaining points of design of the drum are determined from practical considerations and judgment of appearance. The ribs behind the arms are put in to give lateral stiffness and guard against endwise collapse. The arms are subject to the same bending as those of the gear, but as they are equally heavy it is not necessary to calculate them. The flange at the driving end is of course matched to that already designed for the gear. The rope is intended to be brought through the right-hand end with an easy bend and the standard form of button wedged on to prevent its pulling through.

This drum would probably be cast with its axis vertical, and the driving flange down to secure sound metal at that point. Heavy risers would be left at the other end to secure soundness where the rope is fastened. Drums are often cast with the axis horizontal, but the vertical method is more certain to produce a sound casting. The grooves should be turned from the

solid metal, partly because it is a difficult matter to cast them, but principally because the rope should run on as smooth surface as possible to avoid undue wear. On drums which carry chain instead of wire rope the grooves are sometimes cast with success, although even in this case the turned groove is generally preferable.

The brake consists of a wrought-iron band to which are fastened wooden blocks, the iron band giving the requisite strength while the blocks give frictional grip on the drum surface and can be easily replaced when worn. As in the designing of a belt the object in view is the grip on the pulley surface by the leather to enable power to be transmitted from the belt to the pulley, so in the case of the brake if we put the proper tension in the strap it can be made to grip the brake drum so tightly that motion between it and the drum cannot occur. The latter case is really the reverse of the first, if the driven pulley be considered, but is identical with the case of the driving pulley, in which the power is transmitted from the pulley to the belt. Of course in the case of the brake no power is transmitted, as when the brake holds no motion occurs, but the principle of the relative tensions in the strap is the same as for the belt.

Since the brake drum surface is 28 inches in diameter, the load at that surface which the brake must hold is

$$P = \frac{5{,}000 \times 27}{14 \times 2} = 4{,}821 \text{ pounds.}$$

We have then the following calculation corresponding exactly to that of the belt given in Fig. 3.

$$\log. \frac{T_n}{T_o} = 2.729 \times \mu \times n \qquad \text{Let } \mu = .25$$
$$\text{“} \quad n = .75$$

$$T_n - T_o = P = 4{,}821$$

$$\log. \frac{T_n}{T_o} = 2.729 \times .25 \times .75 = 0.512 \text{ (for which the natural number is 3.25).}$$

$$\text{Then } \frac{T_n}{T_o} = 3.25 \qquad T_o = \frac{T_n}{3.25}$$

$$T_n - T_o = 4{,}821 \qquad T_n - \frac{T_n}{3.25} = \frac{2.25 \times T_n}{3.25} = 4{,}821$$

$$T_n = \frac{4{,}821 \times 3.25}{2.25} = 6{,}963 \text{ pounds (say 7,000)}$$

$$T_o = 6{,}963 - 4{,}821 = 2{,}142 \text{ pounds (say 2,200)}$$

The tight end of the strap must then be capable of carrying a load of 7,000 pounds, and since the width has already been taken at $4\frac{1}{2}$ inches, the problem is to find the necessary thickness. Equating the external load to the internal resistance we have

$$7{,}000 = \mathrm{A} \times \mathrm{S} \qquad \text{Let } t = \text{thickness}$$
$$\text{" } \mathrm{S} = \text{fibre stress} = 12{,}000$$
$$7{,}000 = 4.5 \times t \times 12{,}000$$
$$t = \frac{7{,}000}{4.5 \times 12{,}000} = .13 \text{ inch}$$

This, however, can be but a preliminary figure, for the riveting of the strap will take out some of the effective area, and the thickness will have to be increased to allow for this. Suppose on the basis of this figure we assume the thickness at a slightly increased value, say $\frac{3}{16}$ inch, and proceed to calculate the rivets.

A group of five rivets will work in well for this case, which gives $\frac{7{,}000}{5} = 1{,}400$ pounds per rivet. A safe shearing fibre stress is 6,000, hence the area necessary per rivet is $\frac{1{,}400}{6{,}000} = .23$ square inch. This comes nearest to the area $\frac{9}{16}$ diameter, but for the sake of using the more general size of rivet ($\frac{5}{8}$ inch) the latter is chosen, for which the area is .30.

We must now try these rivets in a $\frac{3}{16}$-inch plate for their safe bearing value. The projected area of a $\frac{5}{8}$-inch hole in a $\frac{3}{16}$-inch plate is $\frac{5}{8} \times \frac{3}{16} = .117$ square inch. $\frac{1{,}400}{.117} = 11{,}965$ (15,000 would be safe)

Taking out two $\frac{5}{8}$-inch rivets from the full width of $4\frac{1}{2}$ inches leaves $4\frac{1}{2} - (2 \times \frac{5}{8}) = 3.25$, and makes the net area of strap to take stress $3.25 \times \frac{3}{16} = .61$ square inches. Re-calculating the fibre stress for this area gives

$$7{,}000 = .61 \times \mathrm{S}$$
$$\mathrm{S} = \frac{7{,}000}{.61} = 11{,}475$$ (which approximates the previous value of 12,000).

The slack end of the strap has to take but 2,200 pounds, hence a different calculation might be made for this end giving smaller rivets; but as it is impractical to change the thickness of the strap to meet this reduced load, it is well to maintain the same proportion of joint as at the tight end. The spacing of the rivets in both

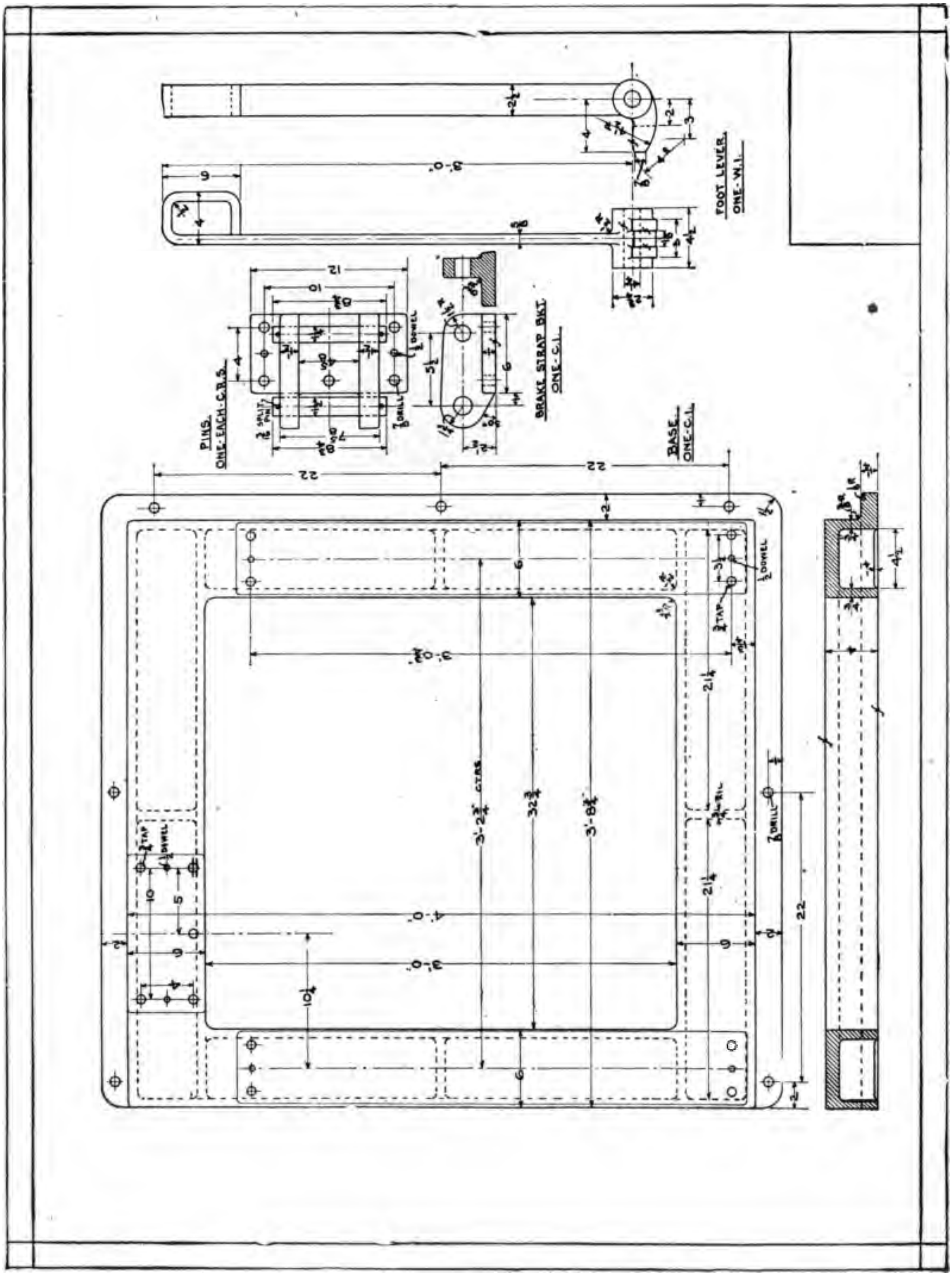

FOOT LEVER.
ONE-W.I.
PINS.
ONE-EACH-C.R.S.
BRAKE STRAP BKT.
ONE-C.I.
BASE.
ONE-C.I.

Fig. 16.

cases follows the ordinary rule allowing at least three times the diameter of the rivet as center distance, and one-half this value to the edge of the plate.

The threaded end of the forging on the strap also has to carry the load of 2,200 pounds, for which a size smaller than 1 inch would suffice. It is natural, however, for the sake of general proportion to make the bolt as strong as the strap, and a 1-inch bolt gives an area of .52 square inch, nearly equalling the value of .61 net area of strap noted above.

Base, Brake-Strap Bracket and Foot Lever. Fig 16. The base cannot be definitely calculated, and can best be proportioned by judgment. It must not distort, twist, or spring in any way to throw the shafts out of line. The area in contact with the foundation upon which it rests must be ample to carry the weight of the whole machine with a low unit pressure. Although the form shown is perfectly practicable to cast and machine, and is simple and rigid, yet it is questionable if a bolted-up construction, say of four pieces, might not be equally rigid and yet involve greater facility of production in both the foundry and machine shop on account of the reduced sizes of parts to be handled. This is a question which depends on the equipment and methods of the individual shop, and is an illustration of the practical control of design by manufacturing conditions.

The brake-strap bracket and foot lever, also shown in this figure, are examples of machine parts which are quite definitely loaded, and the designing of which is a simple matter. Further discussion of their design is not made, the student being given opportunity for some original thought in determining the forces and moments that control their design.

Gear Guard and Brake-Relief Spring. In exposed machinery of this character it is desirable to cover over the gears with a guard to prevent anything accidentally dropping between the teeth and perhaps wrecking the whole machine. This guard is not shown, as it involves little of an engineering nature to interest the student. It could readily be made of sheet metal or light boiler plate, bent to follow the contour of the gears and fastened to the top flange of the main bracket.

If the brake be not automatically supported at its top it will

lie with considerable pressure, due to its own weight, on the brake surface when it is supposed to be free from it, and by the friction thereby created will produce a heavy drag and waste of power. A spring connection fastened to an overhead beam is a simple way of accomplishing the desired result. A flat supporting strap carried out from the gear guard, having some degree of spring in it, is a neater method of solving the problem. The spring should be just strong enough to counterbalance the weight of the strap and yet not resist to an appreciable degree the force applied to throw the brake on.

GENERAL DRAWING.

The last step in the process of design of a machine is the making of the assembled or general drawing. This should be built up piece by piece from the detail drawings, thereby serving as a last check on the parts going together. This drawing may be a cross section or an outside view. In any case it is not wise to try to show too much of the inside construction by dotted lines, for if this be attempted, the drawing soon loses its character of clearness, and becomes practically useless. A general drawing should clearly **hint at, but not specify,** detailed design. It is just as valuable a part of the design as the detail drawing, but it cannot be made to answer for both with any degree of success. A good general drawing has plenty of views, and an abundance of cross sections, but few dotted lines.

The general drawing of the machine under consideration is left for the student to work up from the complete details shown. It would look something like the preliminary layout of Fig. 10, if the same were carefully carried out to finished form. A plain outside view would probably be more satisfactory in this case than a cross section, as the latter would show little more of value than the former. The functions which the general drawing may serve are many and varied. Its principal usefulness is, perhaps, in showing to the workman how the various parts go together, enabling him to sort out readily the finished detail parts and assemble them, finally producing the complete structure. Otherwise the making of a machine, even with the parts all at hand, would be like the putting together of the many parts of an intricate puzzle, and much time

would be wasted in trying to make the several parts fit, with perhaps never complete success in giving each its absolutely correct location.

The general drawing also gives valuable information as to the total space occupied by the completed machine, enabling its location in a crowded manufacturing plant to be planned for, its connection to the main driving element arranged, and its convenience of operation studied.

In some classes of work it is a convenient practice to letter each part on the general drawing, and to note the same letters on the specification or order sheet, thus enabling the whole machine to be ordered from the general drawings. This is a very excellent service performed by the general drawing in certain lines of work, but for such a purpose the drawing is quite inapplicable in others.

Merely as a basis for judgment of design, the general drawing fulfils an important function in any class of work, for it approaches the nearest possible to the actual appearance that the machine will have when finished. A good general drawing is, for critical purposes, of as much value to the expert eye of the mechanical engineer as the elaborate and colored sketch of the architect is to the house builder or landscape designer.

From the above it is readily understood that the general drawing, although a mere putting together of parts in illustration, is yet of great assistance in producing finished and exact machine design.

GENERAL COMMENTS ON PRECEDING PROBLEM.

After following through the detail of work as given in the preceding pages, it is worth while to stop for a moment and take a brief survey or review of the subject as illustrated therein.

If the text be carefully studied it will be seen that in every part to be designed the same routine method has been followed, regardless of the final outcome. In some cases it may seem a roundabout procedure to follow a train of thought that finally ends in a design apparently based on purely practical judgment, the theory having had but very little if any influence. The question at once arises—Why not use the empirical rule or formula in

the first place? Why not make a good guess at once? Why not save all the time and energy devoted to a careful analysis and theory, if we are finally to throw them away and not base our design on them?

The principle to be noted in this connection is, that it is just as fatal to good design to rely upon bare experience and upon judgment alone, as it is to construct solely according to what pure theory tells us. There are many things in the operation of machinery that are totally inexplicable from the purely practical point of view, and will forever remain so until we analyze them and theorize on them. Many good things in machinery have been the result of what might be called "reversed" machine design. When a new machine is started, it frequently, or we might almost say always, fails to do its work just as it is expected to do it. This is because some little point of design is bad, owing to the inability of drawings, however good they may be, to show all that the machine itself in bodily form and in motion shows.

Now, if our analysis and theory have been good in the designing process, it is almost sure that we can very readily analyze and theorize on the trouble that exists when the machine is finished, can detect the weakness, and can correct it with comparatively small change in the general design. This is "reversed" machine design.

If, on the contrary, we have based our design purely on guesswork, allowing our fancy full and free play to work out the details without further basis, we may consider ourselves lucky if the machine runs at all. This, however, is not the worst of the situation. If the machine does actually operate, even as well as it might reasonably be expected to, but still has the usual difficulty of some little kink or hitch that was not expected, then, as a result of the method upon which the whole thing has been constructed, we have no definite plan of action to proceed upon. We must try first this, then that scheme to obviate the trouble. We may be fortunate enough to "strike it" the first time; we may never strike it. It is doubtful if the machine ever can be made to work at highest efficiency; and if fairly good results be finally obtained we never know the reason why, and have nothing on which to base any future action or design.

This haphazard process is not machine design at all, either in name or in result.

As has previously been stated in these pages, there is no such thing as too much analysis or theory in the designing of machinery. Even if we carefully analyze, theorize with rigorous exactness, and then practically modify our construction to such a point that the original theoretical shape is almost or entirely lost, the apparently roundabout process is not in vain, for we are in perfect control of our design. We know exactly what it has to take in the way of forces, blows and vibrations. We know what its ideal shape should be. We know where we can practically modify its form without weakening it excessively or adding excess of material. In other words we know all about it, and therefore know exactly what we can do with it ; and whether it follows in its shape the outline that pure theory gives it or some other outline, it is nevertheless well designed.

"Reversed" machine design, as described above, based on observation and experiment with regard to machines already in operation, is just as impossible without exact analysis and theory as is original design based merely on mechanical ideas in the abstract. The method once learned and made a habit of mind will produce results with equal facility in either case, and results are what the mechanical world is seeking.

Another point worth noting in the progress of the problem as given is the absolute necessity of possessing some knowlege of Mechanics. The more of this subject the designer can have at his finger ends, the more ready and successful will he be in all problems of Machine Design. However, the principles of forces and moments clearly understood, and the application of the same in the all-important subject, "Strength of Beams," constitute a fund of information that will give a splendid start and a good working basis for simple designs. It should always be remembered that a complicated design is little more than a combination of simple designs, and if one has the ability to dissect and analyze what seems at first like a bewildering maze of parts, complication is speedily changed to simplicity.

Common sense goes a long way in good designing. There is nothing mysterious about the process If the beginner will only

avoid doing things that are foolish and ridiculous on their very face, if he will exercise the same judgment that he uses in the daily affairs of his life and will mix in something of mechanics and mechanical method, he will be on the direct road to success in the art.

Good drawing is an essential element of good design, and it is especially urged that the sketches and drawings as reproduced in the preceding text be studied with this in mind. By a good drawing is meant not a showy piece of work, finely shaded or artistically lettered, but an exact layout, definite and measurable, correctly dimensioned if in detail, and meaning exactly what it says. Machine design is an exact science, and the designer cannot shirk responsibility by permitting his work to be shiftless and loose. If he cannot delineate clearly and in definite form what he determines in his mind the structure should be, then it is purely good luck if he achieves success, and it may safely be asserted that the success is due to some subsequent care and finished design added to his feeble effort, rather than to any expertness of his own. Such success is of a very doubtful nature, and if not bordering on financial loss it is at least secured only at a low working efficiency.

As examples of good drawings the plates shown are not claimed to be anything extraordinary, but it will be noted that they are clean-cut and definite, and that even the sketches are unmistakable as to that which they are intended to illustrate. The information as to the design is all there; nothing is left to the imagination.

Classification of Machinery. It is intended to be made clear in all that has preceded, that the same method of attack and procedure may be applied to the designing of machinery, whatever may be the class or kind. This is a fundamental principle. When it is logically carried out, however, it produces very different results, as is evidenced by the characteristics of style peculiar to each of the classes of machinery to one or another of which all machines belong.

For example, an engine lathe has a style similar to a drill press, or a boring mill, or a screw machine, or a milling machine. It is very different, however, from the style of a steam engine, or a pump, or an air compressor, or a locomotive; it is still more dif-

ferent from the style of a rolling mill, or a link belt conveyor, or a coal crusher, or a stamp mill.

These classes of machinery are so distinctly marked that the novice is easily able to perceive that there is some controlling influence in each which marks its peculiar style. He should at the same time see that the very analysis that has been so strongly insisted upon in these pages is the direct cause of the marked characteristic in design. Each class of machinery must satisfy certain exacting conditions different from those of any other, and it is the careful study of these conditions, as fundamentally enforced, which leads to the strictly logical design.

A few of the most common classes are enumerated below, and their prominent features noted. It is hoped that a study of them will familiarize the student in a general way with the requirements of each, and serve as a guide to a more comprehensive study of their detail design than is possible in these pages.

Machine Tools. Examples:—lathe, planer, milling machine, drill press, screw machine, boring mill, grinding machine, etc., etc.

The machines of this class are all utilized for the finishing of metal surfaces. They are really at the root of the production of machinery of all other classes. Accuracy is their prime characteristic—accuracy of construction, accuracy of operation, accuracy of adjustment. Any inaccuracy that exists primarily in a machine tool is reproduced in every piece upon which it produces a finished surface; and since the mere act of finishing a surface upon anything implies that a rough and inaccurate surface will not answer, the tool then fails of its purpose if it cannot produce a true surface: it does not accomplish that for which it was designed.

The effect that this element of accuracy has upon the design of a machine tool is to require long bearings, convenient and exact methods of adjustment, stiffness, excess of material to absorb vibration, special shapes to facilitate application of jigs, fixtures, and exact manufacturing devices insuring interchangeability of parts, dust guards, and automatic lubrication.

Machine tools are essentially machines of maximum output, and depend for their success, not only upon their accuracy as noted, but also upon their ability to do the greatest amount of work per square foot of space occupied, with the least amount of manual

labor and attention on the part of the operator. This is especially true of automatic machinery, which perhaps might be classed by itself in this respect, but which is nevertheless included under the broad term of a machine for producing finished surfaces, being merely the highest and most refined form of same. For machines of this class the designer has to study every detail with the most minute attention, packing away the operating parts into the smallest space and yet providing ready means for access, removal, and repair. Clearances that would be too little for other kinds of machinery are permitted and provided for; material of high grade, strength, and wearing quality, though expensive in first cost, and requiring the most expert skill to finish and to fit into place, must be used in order to keep the machine compact and yet of large capacity, to make it reasonably light in weight and yet amply strong.

Another point which has a great influence on the design of a machine tool is that we can never tell in advance just what it will have to stand in work, for the variation in the material that it finishes, the uncertain skill of the operator who runs it, the crowding to its limit of capacity and even beyond in times of press of business, and the many other stresses that may suddenly and without warning be thrown upon it, must all be thought of and provided for.

The points above mentioned are but a few of those which the designer of machine tools has to meet, and are presented merely as illustrations to show the special skill required in this class of machinery. It is readily seen that while the machine tool designer has great latitude in choice of material and in expenditure of money for refinement of structure—perhaps greater latitude than in any other class, yet he is held down as in no other to the final productive results, a small percentage of failure entirely throwing out the machine as a marketable product.

The style and external appearance of machine tools have a character of their own resulting from this extreme detailed care in design. Corners and fillets are carefully rounded; surfaces and intersections are definitely made; in short, the mechanical beauty of a machine tool is seen only from a near view and close inspection, and it is to this end that the design is constantly directed Appearance is a large factor in the sale of a fine tool, and the

prestige of the American trade abroad in this respect is very noticeable.

Motive-Power Machinery. Examples:—Steam engine, gas engine, air compressor, steam pump, hydraulic machinery, etc., etc.

The element of heat enters into the design of all machinery in this class. The natural agents, air, gas, and water, in their various forms, are taken into the machine in the most efficient form in which it is possible to obtain them, are robbed of their energy to provide power, and are discharged in a form as weak and inert as the efficiency of the machine will determine.

In contrast to the class of machinery just studied, it should be noted that these machines do not produce any material thing; that is, they do not produce finished surfaces on metals, make screws or bolts, bore holes in castings, or turn line shafting. They merely take the energy of the natural agent, which is not in a form available for use, and transform it into motive power for general use.

Hence the element of accuracy as entering into the design of these machines is necessary only for their own efficient operation, and not for the quality of the thing which they produce, as in the case of machine tools. For example, the power furnished by one steam engine to drive a line shaft is as good as that of another as far as the rotating of the shaft is concerned, provided, of course, that both are equipped with the same quality of governing mechanism. The fact that one of the engines has a good adjusting device on the main bearing while the other has not is of no consequence from the standpoint of the line shaft, but it is, of course, of consequence respecting the efficient operation of the engines.

The design of steam engines and similar machines is of a rough nature compared with that of machine tools, as far as the detail of surface is concerned. General accuracy is nevertheless essential for the machine's own sake, but while in the machine tool we deal with thousandths of an inch, in the steam engine hundredths of an inch indicates fine work.

These machines are subject to extremes of temperature that have to be provided for in the design and arrangement of the parts. Being prime movers, controlling the operation of many machines, they must be certain to run during their period of work; hence

design and adjustment must be positive, and when the latter cannot be made while running, it must be quickly and definitely accomplished when a stop is made. Simplicity of construction is essential, facilitating cheap and quick repairs. The design should be such that constant attention while running is avoided, the usual attention of the engineer being a safeguard rather than an implied factor of the original design. General rigidity and stiffness are important, also good balancing of the moving parts, and weight for absorption of vibration; otherwise under the constant daily run the machines will tear to pieces not only themselves but their foundations.

As far as external appearance goes in this and subsequent classes to be mentioned we are on a very different basis from that of machine tools. *General* mechanical symmetry of form is aimed at in the design, and the several smaller parts depend for their outline (aside from considerations of strength, which are, of course, always in order) upon the harmonious relation which they bear to the main and fundamental elements of the machine. Such machinery as air compressors, steam engines, pumps, and the like are viewed as a whole, and criticised, not detail by detail, as is the machine tool, but as to general effect of outline observed from some distance. To convey the desired effect to the eye the design must be bold and massive, connections simple and direct, and the smaller parts must not be so dwarfed in size as to appear like delicate ornaments instead of integral parts of the machine. The lines of connected parts must be continuous from one part to the other; and when interrupted by flanges, bosses, or lugs, the latter, which are merely incidental to the former must not be allowed to obscure wholly the main lines of the fundamental pieces.

It is attention to such points as these that marks the difference between well-designed motive-power machinery and that of the opposite character. Even though the little details of fillets and corners and surfaces may have their effect from a close point of view, the design will stand or fall in excellence on its bolder features, as noted above.

Structural Machinery. Examples:—Hoists, cranes, elevators, transfer tables, locomotives, cars, conveyors, cable-ways, etc., etc.

In the two preceding classes that have been noted, cast iron

in the form of foundry castings enters as the principal material. Steel is utilized for shafts, studs, pins, and keys. Also special forgings, malleable iron and steel castings enter as factors in the production of the machinery discussed. Foundry castings, however, compose the great body of the material used, and the chief problems involved are those of the expert moulding of cast iron, and the handling and finishing of the same. For the operating parts, steel of fine grade is used in highly finished form, expensive because of its fineness, and yet a necessity to the extent it is used. Brass and bronze are used in the same way, generally in connection with the bearings for the shafts.

Structural machinery, on the contrary, uses steel as the basis of its construction. The fundamental structure is built up of plates, channels, beams, and angles; castings, though numerous, are relatively small, being riveted or bolted to the main structure and controlled in their design by its requirements.

Steel is used in this manner partly because the exclusive use of castings is prohibited on account of the excessive weight, and therefore expense, and partly because castings could not be made which would possess the necessary toughness and strength. In many cases the size of the machinery is such that castings, even if they could be made, would not support their own weight. Moreover, machinery of this class is subjected to rough service, and yet must be practically infallible under all conditions, neither being uncertain in operation at critical moments nor entirely failing under an extraordinary load.

The design of structural machinery is tied up to conditions existing largely outside of the locality in which the machinery is built. The steel plates and structural shapes required, being products of the rolling mill, have to conform to the latter's standards. The rivets, bolts and other fastenings have to be in accordance with the established practice of the structural iron worker, in order to permit punching, shearing and bending machinery of regular form to be utilized. Shipment on standard railway cars has to be considered, the design often requiring to be modified to permit this and nevertheless insure positive and accurate assembling in the field.

Steel castings, both large and small, find ready application in

this class of work; also steel forgings, requiring to be worked under a heavy hammer and in many cases by specially devised processes.

In structural design less of the actual process of manufacture is under the eye of the designer than in the former classes of machinery which have been considered, and hence more allowance has to be made for things not coming exactly right to the fraction of an inch. It would be bad design to plan any structural piece of work with the same closeness of detail permitted, and in fact required, in the case of machine tools, or even in the case of motive-power machinery. In planning structural work the idea must be carried out, of certainty of operation in spite of roughness of detail and variations of construction. This does not necessarily imply inaccuracy, or shiftless, loosely constructed machinery; on the contrary, quite the reverse. The locomotive, for example, is one of the most refined pieces of mechanism that exists today; and yet the methods applied to the construction of machine tools would prove a failure on the locomotive. The design of a car axle box has to be just right else it will heat and destroy itself; the same is true of the spindle of a fine engine lathe; and yet how rough the former is compared with the latter, and how unsuited either would be for use on the service of the other.

As a general rule structural machinery can be more closely proportioned to theoretically calculated size than can the preceding types. The rolled material of which it is made is of a uniform and homogeneous nature owing to its process of manufacture, hence its every fibre may be counted on to sustain its share of the total load imposed upon it. This is in sharp contrast to the case of cast iron, which is of such a porous and irregular structure that we have to use a large factor of safety to cover this inherent defect.

Steel castings of both small and large size (which are quite apt to be utilized in this class of machinery for parts that can with difficulty be made out of rolled material), if properly designed of uniform thickness, with all corners well filleted and with the channels for the flow of the molten metal direct and ample, are nearly as reliable as rolled steel. In parts subject to excessive vibration, shocks, and sudden wrenchings, as, for example, the

side frames or the connecting rod of a locomotive, the forged and hammered material is practically a necessity. This is especially the case when the possible breakage of the part would cause serious consequences involving heavy loss of life and property.

From the several points of view as above considered, it can be readily appreciated that, while structural work is in one sense rough and unpolished, yet it requires, from an engineering standpoint, quite as much breadth of experience and judgment as any of the other types. The fine-tool designer, least of all, perhaps, requires book theory, but does require an extended machine-shop experience. The designer of motive-power machinery needs pure physical theory and shop experience of a large and broad scope. The structural designer is least of all concerned with refined and minute finishing processes, but utilizes his theory absolutely, even though roughly.

Mill and Plant Machinery. Examples:—Rolling mills, mining machinery, crushers, stamps, rock drills, coal cutters, the machinery of blast furnaces and steel mills, tube mills, etc., etc.

This machinery constitutes a class which in the roughness of its operation exceeds all others. Moreover, it is machinery which for the most part is in continuous operation—24 hours per day and 365 days in the year. Hence refinement, even such as might be permitted in the preceding class of Structural Machinery, would be fatal here. The conditions that surround plant machinery are unfavorable in the extreme to the life of any material or metal, and it is not possible to change these conditions or give more than partial protection to the operating parts. Hence the design of such machinery must proceed primarily on the assumption that abuse and neglect, grinding away of surfaces, chemical eating away of metal, flooding of parts with water gritty and corrosive, subjection to sudden bursts of flame and intense heat, etc., will in a relatively short time totally destroy, perhaps, the entire structure.

In view of the continuous nature of the working process, which must be kept up in spite of these almost insurmountable conditions, the problem in each case becomes one of expediency; and the designs and arrangement of machinery must be so worked out that operation, repair, construction, and installation can all go

on simultaneously without stopping the continuous process, and with but a small degree of inconvenience to the operation of the plant.

This problem, difficult though it may seem, can be worked out successfully, as is evidenced by the great number of plants of the continuous character operating at high efficiency throughout the world. The engineering and designing skill required to accomplish this, is perhaps of the highest degree met with in modern practice, for in it is involved a working knowledge of the possibilities, if not the detailed designs of machinery included in all classes. And yet, as in the most elementary case of simple design that can be conceived, the result is accomplished in the same way, namely, by studying the conditions (analysis), developing an ideal application to those conditions (theory), and then reducing the ideal design to a practical basis (modification).

A Few Pointed Suggestions on Original Design. Original design deals with the development of original mechanical ideas. The prime requisite for the development of an idea is to understand thoroughly the idea in the rough. See distinctly the mark aimed at, and never lose sight of it. If a method of reaching it is already outlined, understand that also thoroughly and the principles involved. It is impossible to go ahead blindly and hope to come out right. No good machine was ever built that does not stand for hours of concentrated thought on the part of its designer. Good machines never *happen*, they always *grow*.

Just as soon as the object to be accomplished is clearly understood, begin to produce some visible work on the problem. Sketch something. Get some ideas on paper. Ideas on paper suggest other ideas. If the problem, for example, is one of lathe design, sketch a rectangle, and call it the headstock; another rectangle, and call it the footstock; a couple of scratches for the centers; some steps for the cone pulley; three or four lines for the bed; and as many more for the supports. There is now something on paper to look at; the design is begun.

It is much better to stare at this sketch, than into blank space trying to imagine the finished design. No matter how rough the sketch may be, a short study of it will develop some limiting conditions that before were not apparent. Guess at a few rough

dimensions; put them on the sketch; develop another view—a plan or a side elevation—all still in the roughest style, without any regard to finished detail. Information will be growing all the while, and the problem will be opening up. At this stage it is probable that the sketch can easily be seen to be wrong in many respects. Perhaps the arrangement will not do at all.

This is a good sign. It shows that the design is progressing. It is a valuable thing to know that certain plans *cannot* be followed. Do not rub out part of the sketch already made and try to remedy it. Begin again. Make another sketch. Sketch paper is cheap. By and by it may prove to be very desirable to have that first rough outline available for comparison; or it may be that some of its ideas can be applied on other sketches. The second sketch may "show up" little or no better than the first. Make another, and another, and another, until the subject is thoroughly digested. It is wonderful how helpful it is to have some marks on paper relative to a design, even though they be of the utmost crudeness. They save imaginative power tremendously; and, even with them, all available powers of imagination will be needed before the design is perfected.

A careful comparison of one's sketches, rejecting here, and approving there, will, little by little, bring about a definite opinion, and the scale drawing can be begun.

As in the case of the first sketch, so in the case of the first scale drawing, get some lines on paper as quickly as possible. Draw something, even if it is nothing more than a straight horizontal line. Do not stare at blank paper for an hour trying to imagine how the tenth or eleventh line is going to be drawn in relation to the first line. Do not worry about the later lines until it is time for them Draw the first line at once; and, when the second line is drawn, if the first line proves to be wrong, make it right. As in the rough sketch, that first horizontal line is an immense relief from the great waste of blank paper of a fresh sheet. It is something to look at. It is the beginning of a detailed design. If it happens not to be the absolutely correct foundation to build upon, it at least is something to tear down. The main purpose of these preliminary drawings is to keep the mind active on the problem; and advance toward the final accom-

plishment of the design is often made quite as rapidly by discovering what to tear down as by consistently building up.

When a detail draftsman who has been used to having all his work laid out for him by an expert designer attempts to take up original work for himself, he encounters the drawing of that first line in a way he never did before. He is apt to worry for some time over the possible or impossible results of drawing that first line. If he continue this, he will be sure to fail. The second line is much easier to draw than the first, and the third than the second; and the next hundred will follow on in comparatively smooth sequence, all because of bold action on the first few lines.

And yet, just as the design appears to be progressing smoothly, and the advanced progress of the drawing seems cause for congratulation, careful consideration may disclose a "snag" not previously known to exist in the problem. Further study pursued along the line of this new discovery may show that the whole layout thus far has been radically wrong; and that a fresh start will have to be made. At such a time the young designer is apt to feel that his labor has all been thrown away, and he becomes discouraged. There is, however, no cause for discouragement. Machine Design might almost be defined to be the "successful elimination of snags." It takes some ability to discover an obstacle of this sort; to know a "snag" when an opportunity to see it is given. It takes a good designer to eliminate such a difficulty after it has been found. If there were no "snags" it would not require great ability to design machines. Many machines fail because in them there are a lot of undiscovered "snags." Others fail because the "snags," although discovered, were not eliminated by careful design.

Do not be afraid to make a lot of "first" drawings. It is just as important to digest the design thoroughly by means of scale drawings, as it was to digest it originally by means of the rough sketches. An attempt to make the first drawing of an original design absolutely right would, it is safe to say, produce a poor design, one that could be much improved by further trial. Let the drawings multiply, one after another, until the final one is reached, in which the perfection of detail will eliminate all the bad points of the preceding drafts and incorporate good ones of its own based on the study of the others.

And yet it is often true that the first design laid out, even after many others have been developed, may be found to possess features that render a return to it desirable. This is why it is always better to produce a collection of designs than to attempt to rub out and work over the first one. The best designers usually have a great number of sketches showing how to accomplish a single result. Likewise, they also have a series of layouts to scale, showing in detailed form the development of their various ideas. This is because, without a careful consideration of many methods, they themselves feel incompetent to judge of the best design possible for accomplishing a given result.

Sketches and original designs should always be dated and signed. Different designers may be working on the same problem, and priority of design will never be allowed except upon signed and witnessed papers. It is embarrassing to find, after months and perhaps years have passed since an original drawing was made, that one's rights have been preëmpted merely because there was no date or signature to define them.

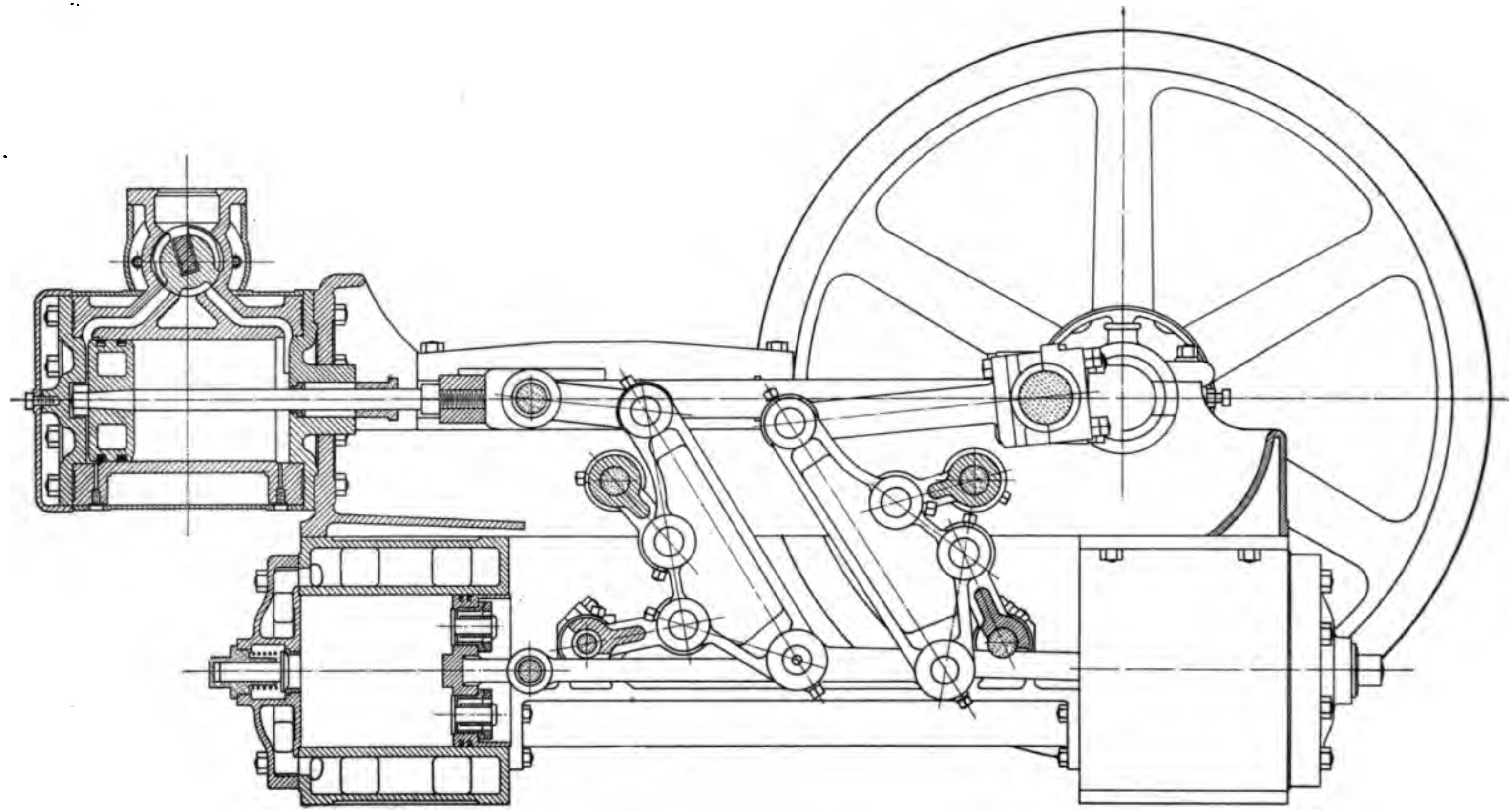
In redesigning or modifying an existing machine, never make a change merely for the sake of doing so. Give the good points of the machine a chance, and devote attention in the new design to correcting the bad points. It is in bad taste, if it be not actually childish, to "look wise and suggest a change" in details which happen to have been designed by another party, but which, nevertheless, are by common engineering judgment pronounced good for the special work intended. This element of unfair and selfish criticism has more than a moral bearing. When it is carried into the superintendence of designing work, it extinguishes the personality of the subordinate draftsman; his efficiency as an original thinker is lowered; and narrow designs are produced.

"The best way for a subordinate to dispose of what appears to be a poor suggestion from a superior, is to work it out to the best degree possible." If it turns out to be good the credit of working it out belongs to the man who did it. If it is actually bad, a careful working out will usually develop the fact beyond dispute, and save unprofitable argument. For the success or failure of a machine there is only one argument better than the detail drawings, and that is the machine itself in operation.

Detail drawings, however, are infinitely better prosecutors or defendants than a multitude of wordy counsel.

Summary. The above classification of machinery might be subdivided and extended indefinitely, and on the broad basis on which it is given it doubtless does not cover the entire field. As an illustration, however, not only of types of machinery, but of methods of design and study, it is hoped that it may be of assistance in giving a start to the student of machine design, in whatever class his interests may happen to lie.

It is the general principles of the art which it is important to master. It is not the designing of a locomotive, or a stationary steam engine, or a crane, or an engine lathe, or a rolling mill, which should be sought to be learned, but the designing of *anything* that may confront us. Specializing is sure to come to the designer in the course of his experience, and when it does he merely fits to the particular specialty the principles he knows for all, and practically develops them along that individual line.



AIR COMPRESSOR (N. Y. AIR BRAKE CO.)
General or Assembled Drawing

any belt however small, if the belt were only laced up tight enough.

This conclusion is literally true; but the important fact now comes in, that the strength of the material of which the belt is made is limited, and while theoretically we might be able to accomplish the above, it would be impossible to do so in practice, for at a certain point the belt would break under the strain. Other practical considerations also come in, which fix this limit of power transmission at a point far below the breaking strength of the material.

The complete analysis is not quite as simple as the above, especially for high-speed belts. When the driving side of the belt becomes tight, it stretches and grows longer; and at the same time the other side of the belt becomes slack and grows shorter. But it is not true that the increase in the one side is the same as the decrease in the other, and this fact produces the condition that the sum of the tensions in motion is not quite the same as the sum of the tensions at rest.

Again, when the belt, as it passes around the pulley, changes its straight-line direction to circular motion, each particle of the belt—like a body whirling at the end of a cord about a center of rotation—tends by centrifugal force to fly away from the surface of the pulley, thereby decreasing the normal pressure, and hence the friction. This centrifugal force also changes somewhat the tensions in the belt between the pulleys. As the centrifugal force increases in proportion to the square of the linear velocity; it is evident that the effect is greater at high speeds than at moderate or low speeds.

A further circumstance that affects the driving power of a belt is the stiffness of the leather or other material of which the belt is made. As it passes around the pulley, the belt is bent to conform to the circumference of the pulley, and is again straightened out as it leaves the pulley. Hence the theoretically perfect action is modified somewhat according to the sharpness of the bending and the thickness or flexibility of the belt; in other words, a small pulley carrying a thick belt would be the worst case for successful calculation on a theoretical basis.

THEORY. The condition of the tight and loose sides of a

belt transmitting power, is similar to that of the weighted strap and fixed pulley shown in Fig. 17. If motion is desired of the strap around the pulley, it is necessary to make the weight W_2 of such a magnitude that it will overcome not only the weight W_1, but also the friction between the strap and the pulley. The strap tension T_n is, of course, equal to W_2, and T_o to W_1. The equation showing the balance of forces for the condition when motion is about to occur, is:

$$T_n - T_o = F = P \text{ (driving force).} \qquad \textbf{(5)}$$

If the pulley be free to turn on its axis, instead of being fixed as in Fig. 17, the strap by its friction on the pulley will turn the pulley, and the force of friction F becomes the driving force for the pulley as noted in equation 5 above.

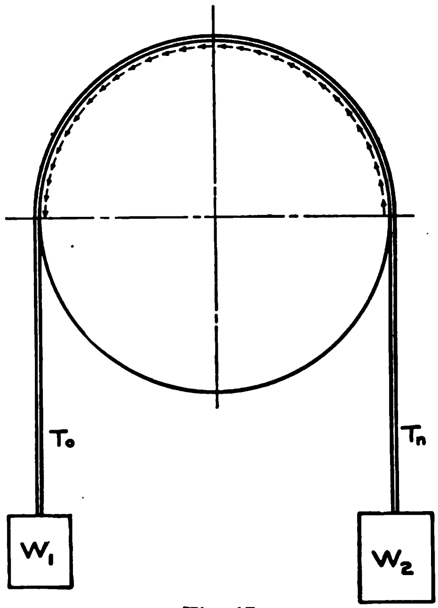


Fig. 17.

In Fig. 18, let us suppose that W is a weight representing the resistance to be overcome. The tensions T_n and T_o, equal at first owing to stretching the belt tightly over the pulleys at rest, change when an attempt is made to raise the weight by turning the larger pulley; and just as the weight leaves the floor, the equality of moments about the axis of the driven pulley gives the following equation:

$$(T_n - T_o)\, r = F \times r = P \times r = W \times r_1. \qquad \textbf{(6)}$$

This equality of moments remains as long as the motion of the weight is uniform, and represents closely the conditions under which belt pulleys work.

Although we know from the above what the *difference* of the belt tensions is, and what this difference will do when applied to

the surface of a given pulley, we do not yet know what either T_n or T_o actually is; and until we do know, we cannot correctly proportion the belt. Hence we must find another relation between T_n and T_o which we can combine with equations 5 and 6. This relation is deduced by a process of higher mathematics, which results as follows:

$$\text{Common logarithm } \frac{T_n}{T_o} = 2.729\ \mu\ (1 - z)n. \quad (7)$$

Treating equations 5 and 7 as simultaneous, values of both T_n and T_o can be found by the regular algebraic solution. As T_n is the larger, the actual area of belt to provide the necessary strength must be made to depend upon it.

The factor z in equation 7 depends upon the centrifugal force

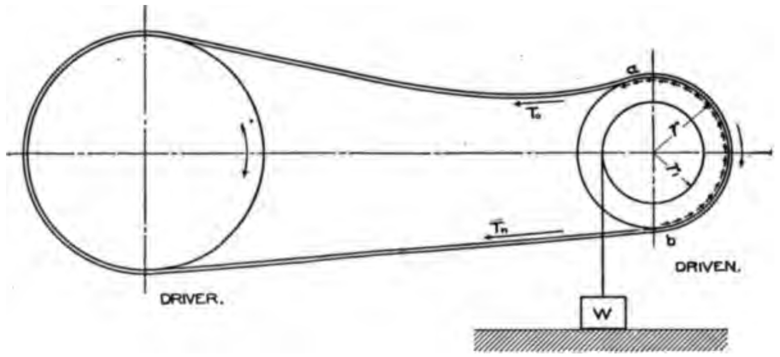


Fig. 18.

developed by the weight of the belt passing around the pulley. Its value, found from mechanics, is:

$$z = \frac{w \times V^2}{9{,}660 \times t}.$$

Having found the maximum pull on the belt, it now remains to write the equation:

$$\text{External force} = \text{Internal resistance};$$

or, $$T_n = b \times h \times t. \quad (8)$$

Usually the most convenient way to handle this equation is to assume h and t, and then solve for b.

Summing up the theoretical treatment of belt design, we simply combine equations 5, 6, 7, and 8, and solve for the quantity desired. Discussion of the constants involved in these equations, and of the practical factors controlling them, is given in the following:

PRACTICAL MODIFICATION. The force of friction F, which is the same as driving force P, depends on:

Coefficient of friction (μ) between belt and pulley;

Tightness of the belt;

Centrifugal force of the belt;

Angle of contact of belt with pulley.

The coefficient of friction (μ), according to experiments and observed operation of belts transmitting power, varies from .15 to .56 for leather on cast iron. An average value consistent with a reasonable amount of slip, the belt being in good running order, is .30. If the belt is oily, or likely to become so in use, a lower value should be taken.

The tighter the belt is drawn up, the greater is the pressure against the pulley, and hence the greater is the force of friction. But if we pull the belt up too tightly, when we begin to drive, T_n becomes too great, and the belt breaks or is under such stress that it wears out quickly. Moreover, the great side pressure on the bearings carrying the shaft produces excessive friction, and the drive is inefficient. This is why a narrow belt driven at high speed is more efficient than a wide belt at slow speed, for we cannot pull up the former as tightly as the latter without overstraining it, and yet it is possible to get the required power out of the narrow belt by running it at high speed.

The centrifugal force is of small importance for low speeds, say of 3,000 feet per minute and less; and it therefore may usually be neglected. The factor z then becomes zero in the expression $1 - z$ in equation 7, and the second member of the equation stands simply $2.729 \times \mu \times n$.

The angle of contact of belt with pulley is important, as a large value gives a great difference between T_n and T_o; and it is desirable to make this difference as great as possible, because thereby the driving force is increased. The loose side of a horizontal belt should always be above, as then the natural sag of the loose

side due to its slackness tends to increase the angle of contact with the pulley, while the tightening up of the lower side acts against its sag to make the loss of wrap as little as possible. Vertical belts which have the driving pulley uppermost, utilize the weight of the belt to increase the pressure against the surface of the pulley, slightly increasing its capacity for driving. The angle of contact may be artificially increased by a tightening pulley which presses the belt further around the pulley than it would naturally lie. It adds however, the friction of its own bearing, and impairs the efficiency of the drive. For ordinary horizontal belts, the angle of contact is but little more than 180°, and the value of n in equation 7 may be safely assumed at $\frac{1}{3}$ unless the pulleys are of relatively great difference of diameter and very close together.

Strength of Leather Belting. The breaking tensile strength of leather belting varies from 3,000 to 5,000 pounds per square inch. Joints are made by lacing, by metal fasteners, or by cementing. The strength of a laced joint may be about $\frac{7}{10}$, of a metal-fastened joint, about $\frac{1}{2}$, and of a cemented joint, about equal to the full strength of the belt cross-sectional area. The proper working strength of belting depends on the use to which the belt is put. A continuously running belt should have a low tension in order to have long life and a minimum loss of time for repairs. For double leather belting it has been shown that a working tension of 240 pounds per square inch of sectional area gives an annual cost — for repairs, maintenance, and renewals — of 14 per cent of first cost. At 400 pounds working tension, the annual expense becomes 37 per cent of first cost. These results apply to belts running continuously; larger values may be used where the full load comes on but a short time, as in the case of dynamos.

Good average values for working tensions of leather belts are:

Cemented joints,	400	pounds per square inch.
Laced joints,	300	" " " "
Metal joints,	250	" " " "

Horse-Power Transmitted by Belting. If P is the driving force in pounds at the rim of the pulley, and V is the velocity of the belt in feet per minute, the theoretical horse-power transmitted is evidently :

$$\text{H. P.} = \frac{\text{P} \times \text{V}}{33{,}000}. \qquad (9)$$

It is evident from the above that the horse-power of a belt depends upon two things, the driving force P and the velocity V. If either of these factors is increased, the horse-power is increased. Increasing P means a tight belt. Hence a tight belt and high speed together give maximum horse-power. But a tight belt means more side strain on shaft and journal. Therefore, from the standpoint of efficiency, *use a narrow belt under low tension at as high a speed as possible.*

Empirical rules for horse-power of belting, if used with judgment, give safe results when applied to very general cases. A common rule used by American engineers is:

$$\text{H. P.} = \frac{b \times \text{V}}{1{,}000}. \qquad (10)$$

For a double belt, assuming double strength, this becomes:

$$\text{H. P.} = \frac{b \times \text{V}}{500}. \qquad (11)$$

With large pulleys and moderate velocities, this may hold good. With small pulleys and high velocities, however, the uncertain stresses induced by the bending of the fibers of the belt around the pulley, and the relatively great loss due to centrifugal force, modify this relation· and a safer value for a double belt of the ordinary kind is:

$$\text{H. P.} = \frac{b \times \text{V}}{540}; \qquad (12)$$

or, still safer,

$$\text{H. P.} = \frac{b \times \text{V}}{700}. \qquad (13)$$

If we compare the theoretical value of equation 9 with the empirical value of equation 10 by putting them equal to each other, thus:

$$\text{H. P.} = \frac{\text{P} \times \text{V}}{33{,}000} = \frac{b \times \text{V}}{1{,}000}.$$

and solve for P, we get :

$$P = 33b. \qquad (14)$$

This develops the fact that the empirical rule of equation 10 assumes a driving force of 33 pounds per inch of width of single belt.

Another way of expressing equation 10 is: A single belt will transmit one horse-power for every inch of width at a belt speed of 1,000 feet per minute.

Speed of Belting. The most economical speed is somewhere between 4,000 and 5,000 feet per minute. Above these values the life of the belt is shortened; also "flapping," "chasing," and centrifugal force cause considerable loss of power. The limit of speed with cast-iron pulleys is fixed at the safe limit for bursting of the rim, which may be taken at one mile per minute.

Material of Belting. Oak-tanned leather, made from the part of the hide which covers the back of the ox, gives the best results for leather belting. The thickness of the leather varies from .18 to .25 inch. It weighs from .03 to .04 pound per cubic inch. The average thickness of double leather belts may be taken as .33 inch, although a variation in thickness from $\frac{1}{4}$ inch to $\frac{7}{16}$ inch is not uncommon. Double leather belts may be ordered light, medium, or heavy.

In a single-thickness belt the grain or hair side should be next to the pulley, for the flesh side is the stronger and is therefore better able to resist the tensile stress due to bending set up where the belt makes and leaves contact with the pulley face. Double leather belts are made by cementing the flesh sides of two thicknesses of belt together, leaving the grain side exposed to surface wear.

Raw hide and semi-raw hide belts have a slightly higher coefficient of friction than ordinary tanned belts. They are useful in damp places. The strength of these belts is about one and one-half times that of tanned leather.

Cotton, cotton-leather, rubber, and leather link belting are some of the forms on the market, each of which is especially adapted to certain uses. For their weights and their tensile and working strengths consult the manufacturers' catalogues.

A prominent manufacturer's practice in regard to the sizes of

leather belting will be found useful for comparison, and is indicated in the table on page 12.

Initial Tension in Belt. On the assumption that the sum of the tensions is unchanged, whether the belt be at rest or driving, we should have the following relation :

$$T_n + T_o = 2T;$$

whence,

$$T = \frac{T_n + T_o}{2}. \qquad (15)$$

This is not strictly true, however, as is stated in the "Analysis" of "Belts." It has been found that in a horizontal belt working at about 400 lbs. tension per square inch on the tight side, and having 2 per cent slip on cast-iron pulleys (*i. e.*, the surface of the

Sizes of Leather Belting.

WIDTH.	THICKNESS.	
	Single.	Double.
1 inch.	$\frac{5}{32}$ inch.	$\frac{5}{16}$ inch.
2 "	$\frac{3}{16}$ "	$\frac{5}{16}$ "
3 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "
4 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "
5 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "
6 "	$\frac{7}{32}$ "	$\frac{3}{8}$ "
10 "	$\frac{5}{16}$ "	$\frac{3}{8}$ "
12 "	....	$\frac{3}{8}$ "
14 "	....	$\frac{13}{32}$ "
20 "	....	$\frac{7}{16}$ "

driven pulley moving 2 per cent slower than that of the driver), the increase of the sum of the tensions when in motion over the sum of the tensions at rest, may be taken at about $\frac{1}{3}$ the value of the tensions at rest. Expressing this in the form of an equation

$$T_n + T_o = \frac{4}{3}(2T) = \frac{8 \times T}{3}.$$

$$T = \frac{3}{8}(T_n + T_o). \qquad (16)$$

The value of T thus found would be the pounds initial tension to which the belt should be pulled up when being laced, in order to produce T_n and T_o when driving.

This value is not of very great practical importance, as the proper tightness of belt is usually secured by trial, by tightening pulleys, by pulley adjustment (as in motor drives), or by shortening the belt from time to time as needed. It is worth noting, however, that for the most economical life of. the belt it would be very desirable in every case to weigh the tension by a spring balance when giving the belt its initial tension. This, however, is not always easy or even feasible; hence it is a refinement with which good practice usually dispenses, except in the case of large and heavy belts.

PROBLEMS ON BELTS.

1. Determine the belt tensions in a laced belt transmitting 50 horse-power at a velocity of 3,500 feet per minute. Suppose that the arc of contact is 180°; weight of belt = .035 pound per cub. in.; and coefficient of friction 25 per cent.

2. What is the width of above belt if it is $\frac{3}{16}$ inch in thickness?

3. What initial tension must be placed on above belt?

4. The main drive pulley of a 120-horse-power water wheel is 6 feet in diameter. A cemented leather belt is to connect the main pulley to a 3-foot pulley on the line shafting in a mill. The horizontal distance between centers of shafting is 24 feet; coefficient of friction, 30 per cent; revolutions per minute of line shafting, 180. Design the belt for this drive.

5. An 8-inch double belt $\frac{3}{8}$ inch thick connects 2 pulleys of 30-inch and 20-inch diameter respectively. The horizontal distance between the centers is 12.5 feet. The coefficient of friction is 0.3, and the weight of belt per cubic inch is 0.035 pound. Working tension, 300 pounds per square inch. Speed of belt 5,000 feet per minute. Lower face of 30-inch pulley is the driving face. Required the H. P. which may be transmitted (theoretically).

6. Compare the theoretical horse-power in problem 5 with that obtained by the use of empirical formula.

PULLEYS.

NOTATION—The following notation is used throughout the chapter on Pulleys:

A = Area of rim (sq. in.).
a = " " arm (" ").
b = Center of pulley to center of belt (inches; practically equal to R).
C_1 = Total centrifugal force of rim (lbs.).
c = Distance from neutral axis to outer fiber (inches).
D = Diameter of pulley (inches).
D_1 = " " hub (").
d_1 = " " bolt at root of thread (inches).
d = Diameter of bolt holes (inches).
g = Acceleration due to gravity (ft. per sec.).
h = Width of arm at any section (inches).
I = Moment of inertia.
L = Length of arm, center of belt to hub (inches).
L_1 = Length of rim flange of split pulley (inches).
l = Length of hub (inches).
N = Number of arms.
n = " " rim bolts, each side.
P = Driving force of belt (lbs.).
P_1 = Force at circumference of shaft (lbs.).
P_2 = Force at circumference of hub (lbs.).
p = Stress in rim due to centrifugal force (lbs. per sq. in.).
R = Radius of pulley (inches).
S = Fiber stress (lbs. per sq. in.).
s = Fiber stress in flange (lbs. per sq. in.).
T = Thickness of web (inches).
t = " " rim (").
t_2 = " " " bolt flange (inches).
T_n = Tension of belt on tight side (lbs.).
T_0 = " " " " loose " (").
v = Velocity of rim (ft. per sec.).
w = Weight of material (lbs. per cub. in.).

ANALYSIS. If a flexible band be wrapped *completely* about a pulley, and a heavy stress be put upon each end of the band, the rim of the pulley will tend to collapse just like a boiler tube with steam pressure on the outside of it. A compressive stress is induced which is very nearly evenly distributed over the cross-section of the rim, except at points where the arms are connected thereto. At these points the arms, acting like rigid posts, take this compressive stress. Now, a pulley never has a belt wrapped *completely* round it, the fraction of the circumference embraced by the belt being usually about $\frac{1}{2}$, and seldom, even with a tightener pulley, reaching $\frac{3}{4}$. Assuming the wrap to be $\frac{1}{2}$ the circumference, and that all the side pull of the belt comes on the rim, none being transmitted through the arms to the hub, we then have one-half of the rim pressed hard against the other half by a force equal to the resultant of the belt tensions, which, in this case, would be the sum of them. Dividing the pulley by a plane through its center and perpendicular to the belt, the cross-section of the rim cut by this plane has to take this compressive stress.

This analysis is satisfactory from an ideal standpoint only, for the intensity of stress due to the direct pull of the belt, with the usual practical proportions of rim, would be very small. Moreover, the element of speed has not been considered.

When the pulley is under speed, a set of conditions which

complicates matters is introduced. The centrifugal force due to the weight of the rim and arms is no longer negligible, but has an important influence upon the design and material used. This centrifugal force acts against the effect of the belt wrap, tending to reduce the compressive stress, or, overcoming the latter entirely, sets up a tensional stress both in the rim and in the arms. It also tends to distort the rim from a true circle by bowing out the rim between the arms, thus producing a bending moment in the rim, maximum at the points where the rim joins each arm.

It can readily be imagined that the analysis in detail of these various stresses in the rim acting in conjunction with each other is quite complicated — far too much so in fact, to be introduced here. As in most cases of such design, however, one controlling influence can be separated out from the others, and the design based thereon with sufficient margin of strength to satisfy the more obscure conditions. This is rational treatment, and the "theory" will be studied accordingly.

The rim, being fastened to the ends of the arms, tends, when driving, to be sheared off, the resisting area being the areas of the cross-sections of the arms at their point of joining the rim. The force that produces this shearing tendency is the driving force of the belt, or the difference between the tensions of the tight and loose sides.

Again, at the point of connection of the arms to the hub, a shearing action takes place, so that, if this shearing tendency were carried to rupture, the hub would literally be torn out of the arms. Now, viewing the arms as beams loaded at the end with the driving force of the belt, and fixed at the hub, a heavy bending stress is set up, which is maximum at the point of connection to the hub. If the rim were stiff enough to distribute this driving force equally between the arms, each arm would take its proportional share of the load. The rim, however, is quite thin and flexible; and it is not safe to assume this perfect distribution. It is usual to consider that one-half the whole number of arms take the full driving force.

THEORY—Pulley Rim. Evidently it is practically impossible to make so thin a rim that it will collapse under the pull of a belt. As far as the *theory* of the rim is concerned, its proportion prob-

ably depends more upon the calculation for centrifugal force than upon anything else.

In order to separate this action from that of any other forces, let us suppose that the rim is entirely free from the arms and hub, and is rotating about its center. Every particle, by centrifugal force, tends to fly radially outward from the center. This condition is represented in Fig. 19. The tendency with which one-half of the rim tends to fly apart from the other is indicated by the force C_1; and the relation between C_1 and the small radial force c for each unit-length of rim can readily be found from the principles of mechanics. The case is exactly like that of a boiler or a thin pipe subjected to uniform internal pressure, which, if carried to rupture, would split the rim along a longitudinal seam.

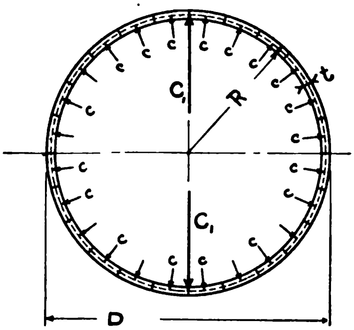


Fig. 19.

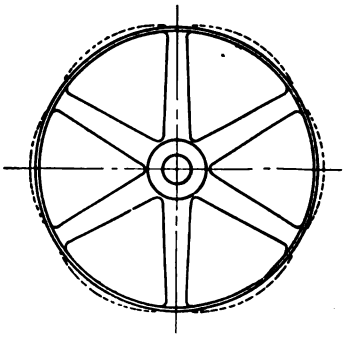


Fig. 20.

The tensile stress thus induced per square inch can be found by simple mechanics to be:

$$p = \frac{12wv^2}{g}; \qquad (17)$$

or, since $w = 0.26$ pound, and $g = 32.2$ feet per second,

$$p = 0.097\,v^2 \quad (\text{say } \frac{v^2}{10}); \qquad (18)$$

and, if p be taken equal to 1,000 pounds per square inch, which is as high as it is safe to work cast iron in this place,

$$v = 100 \text{ feet per second.} \qquad (19)$$

This shows the curious fact that the intensity of stress in the rim

is directly proportional to the square of the linear velocity, and wholly independent of the area of cross-section. It is also to be noted that 100 feet per second is about the limit of speed for cast-iron pulleys to be safe against bursting.

If we wish to consider theoretically the rim together with the arms as actually connected to it, we get a much more complicated relation. This condition is shown in Fig. 20, where the rim, expanding more than the arms, bulges out between them. This makes the rim act something like a continuous beam uniformly loaded; but even then the resulting stress is not clearly defined on account of the variable stretch in the arms. Investigation on this basis is not needed further than to note that it is theoretically better, in the case of a split pulley, to make the joint close to the arms, rather than in the middle of a span.

Pulley Arms. The centrifugal force developed by the rim and arms tends to pull the arms from the hub. On the belt side, this is balanced to some extent by the belt wrap, which tends to compress the arm and relieve the tension. On the side away from the belt, the centrifugal action has full play, but the arm is usually of such cross-section that the intensity of this stress is very low. It may safely be neglected.

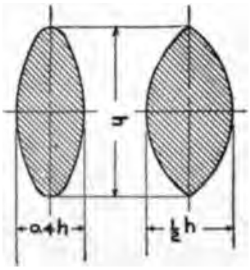


Fig. 21.

The rim being very thin in most cases, its distributing effect cannot be depended on, hence the driving force of the belt may be taken entirely by the arms immediately under the portion of the belt in contact with the pulley face. For a wrap of 180° this means that only one-half of the pulley arms can be considered as effective in transmitting the turning effort to the hub. Each of these arms is a lever fixed at one end to the hub and loaded at the other. A lever of this description is called a "cantilever" beam, its maximum moment existing at its fixed end. The load that each of these beams may be subjected to is $\frac{P}{\frac{N}{2}}$, and therefore the maximum external moment at the hub is $\frac{2PL}{N}$. From mechanics we

know that the internal moment of resistance of any beam section is $\frac{SI}{c}$, and that equilibrium of the beam can be satisfied only when the external moment is equal to the internal moment of resistance of the beam section. Equating these two, we have:

$$\frac{2PL}{N} = \frac{SI}{c}. \tag{20}$$

The arms of a pulley are usually of the elliptical or segmental cross-section, and may be of the proportions shown in Fig. 21. For either of these sections the fraction $\frac{I}{c}$ is approximately equal to $0.0393h^3$. For convenience (the error caused being on the safe side), L may be taken as equal to the full radius of the pulley R, whence

$$\frac{2PR}{N} = \frac{2(T_n - T_o)R}{N} = 0.0393Sh^3, \tag{21}$$

in which S may be from 2,000 to 2,250 for cast iron

Taking moments about the center of the pulley, and solving for P_2, the force acting at the circumference of the hub, we have:

$$\frac{2PR}{N} = \frac{P_2D_1}{2};$$

or

$$P_2 = \frac{4PR}{ND_1} \tag{22}$$

The area of an elliptical section is π times the product of the half axes. With the proportions of Fig. 21, this becomes:

$$a = \pi \times 0.2h \times 0.5h = \frac{\pi h^2}{10} \tag{23}$$

Equating the external force to the internal shearing resistance, we have:

$$\frac{4PR}{ND_1} = \frac{\pi h^2 S_s}{10}$$

or,

$$S_s = \frac{40PR}{D_1N\pi h^2}, \tag{24}$$

in which the shearing stress S_s may run from 1,500 to 1,800 for cast iron.

Although both bending and shearing stresses as calculated above exist at the base of the arms, the bending is, in practically every case, the controlling factor in the design of the arms. An arm-section large enough to resist bending would have a very low intensity of shear.

If the number of arms be increased indefinitely, we come to a continuous arm or web, in which the bending action is eliminated. It may still shear off at the hub, where the area of metal is the least, at minimum circumference. In this case the area under shearing stress is $\pi D_1 T$; and the force at the circumference of the hub, is

$$\frac{PR}{\frac{D_1}{2}} = \frac{2PR}{D_1}$$

Equating external force to internal shearing resistance, we have:

$$\frac{2PR}{D_1} = \pi D_1 T S_s;$$

$$\text{or,} \quad S_s = \frac{2PR}{\pi D_1^2 T}. \qquad (25)$$

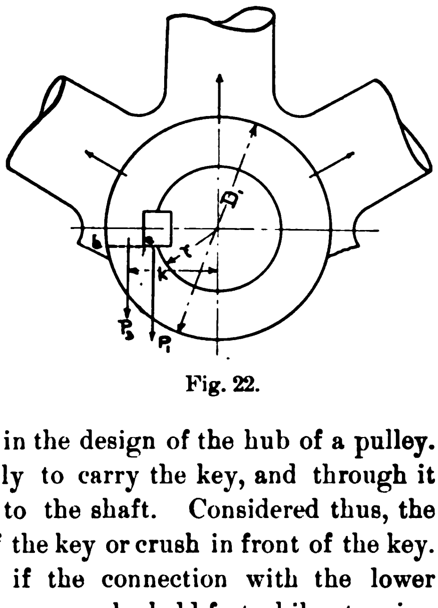


Fig. 22.

Pulley Hub. As in the case of the arms, centrifugal force does not play much part in the design of the hub of a pulley. The hub is designed principally to carry the key, and through it transmit the turning moment to the shaft. Considered thus, the hub may tear along the line of the key or crush in front of the key.

For example, in Fig. 22, if the connection with the lower arms be neglected, and the upper arms be held fast while a turning force P_1, at the surface of the shaft, is transmitted to the hub through the key, then the metal of the hub directly in front of the key is under crushing stress; and the metal along the line *eb*, from the corner to the outside, is under tensile stress. This condition is the worst that could possibly happen, because the bracing effect of the lower arms has been neglected, and the key is located between the arms.

Taking moments about the center of the shaft, the value of the force at the shaft circumference, or the "key pull," is:

$$P_1 = \frac{P\,R.}{r} \qquad (26)$$

Now $\frac{P_1}{P_3} = \frac{k}{r}$, k being the distance from the center of shaft to center of eb, and the area of metal which is subjected to the tearing action P_3 is $l \times eb$. Equating the external force to the internal resistance, and assuming that the stress is equally distributed over the area $l \times eb$, we have:

$$P_3 = \frac{r}{k} P_1 = \frac{r}{k} \times \frac{P\,R}{r} = S \times l \times e\,b;$$

or,

$$S = \frac{PR}{k \times l \times eb}. \qquad (27)$$

The intensity of crushing on the metal in front of the key, due to force P_1, depends upon the thickness of the key, and is properly discussed later under "Keys."

PRACTICAL MODIFICATION—Pulley Rim. The theoretical calculation for the thickness of the rim may give a thickness that could not be cast in the foundry, and the section in that case will have to be increased. As light a section as can be readily cast will usually be found abundantly strong for the forces it has to resist. A minimum thickness at the edge of the rim is about $\frac{3}{16}$ inch; and as the pulleys increase in size, the rim also must be made thicker; otherwise the rim will cool so much more quickly than the arms, that the latter, on cooling, will develop shrinkage cracks at the point of junction.

For a velocity of 6,000 feet per minute, we find from equation 18 that the tension in pounds per square inch, in the rim, due to centrifugal force, is 970. Though this in itself is a low value, yet the uncertain nature of cast iron, its condition of internal stress, due to casting, and the likely existence of hidden flaws and pockets, have established the usage of this figure as the highest safe limit for the peripheral speed of cast-iron pulleys. It is easily remembered that *cast-iron pulleys are safe for a linear velocity of about one mile per minute.*

To prevent the belt from running off the pulley, a "crown" or rounding surface is given the rim. A tapered face, which is more easily produced in the ordinary shop, may be used instead. This taper should be as little as possible, consistent with the belt staying on the pulley; $\frac{1}{2}$ inch per foot each way from the center is not too much for faces 4 inches wide and less; while above this width $\frac{1}{4}$ inch per foot is enough. As little as $\frac{1}{8}$ inch total crown has been found to be sufficient on a 24-inch face, but this is probably too little for general service.

Instead of being "crowned," the pulley may be flanged at the edges; but flanged pulley rims chafe and wear the edge of the belt.

The inside of the rim of a cast-iron pulley should have a taper of $\frac{1}{2}$ inch per foot to permit easy withdrawal from the foundry

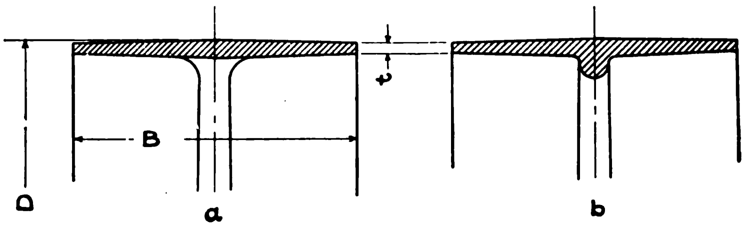


Fig. 23.

mould. This is known as "draft." If the pattern be of metal, or if the pulley be machine-moulded, the greater truth of the casting does not require that the inside of the rim be turned, as the pulley, at low speeds, will be in sufficiently good balance to run smoothly. For roughly moulded pulleys, and for use at high speeds, however, it is necessary that the rim be turned on the inside to give the pulley a running balance.

Fig. 23 shows a plain rim *a* also one stiffened by a rib *b*. Where heavy arms are used this rib is essential so that there will not be too sudden change of section at the junction of rim and arm, and consequent cracks or spongy metal.

Pulley Arms. The arms should be well filletted at both rim and hub, to render the flow of metal free and uniform in the mould. The general proportions of arms and connections to both hub and rim may perhaps be best developed by trial to scale on the drawing board. The base of the arm being determined, it may gradu-

ally taper to the rim, where it takes about the relation of $\frac{2}{3}$ to $\frac{3}{4}$ the dimensions chosen at the hub. The taper may be modified until it looks right, and then the sizes checked for strength.

Six arms are used in the great majority of pulleys. This number not only looks well, but is adapted to the standard three-jawed chucks and common clamping devices found in most shops. Elliptical arms look better than the segmental style. The flat, rectangular arm gives a very clumsy and heavy appearance, and is seldom found except on the very cheapest work.

A double set of arms may be used on an excessively wide face, but it complicates the casting to some extent.

Although a web pulley may be calculated for shear at the hub, yet it will usually be found that with a thickness of web intermediate between the thickness of the rim and that of the hub, which will satisfy the casting requirements, the requirements as to strength will be fully met.

Pulley Hub. The hub should have a taper of $\frac{1}{2}$ inch per foot draft, similar to that of the inside of the rim. The length of the hub is arbitrary, but should be ample to prevent rocking on the shaft. A common rule is to make it about $\frac{3}{4}$ the face width of the pulley.

The diameter of the hub, aside from the theoretical consideration given above, must be sufficient to take the wedging action of a taper key without splitting. This relation cannot well be calculated. Probably the best rule that exists is the familiar one that the hub should be twice the diameter of the shaft. This rule, however, cannot be literally adhered to, as it gives too small hubs for small shafts and too large ones for large shafts. It is always well to locate the key, if possible, underneath an arm instead of between the arms, thus gaining the additional strength due to the backing of the arm.

SPLIT PULLEYS.

ANALYSIS and THEORY. The split pulley is made in halves and provided with bolts through flanges and bosses on the hub for holding the two halves together. When the pulley is in place on the shaft, bolted up as one piece, it is subjected to the same forces as the simple pulley. Hence its general design fol-

lows the same principles, and we need only study the fastening of the two halves, and the effect of this fastening on the detail of rim and hub.

The simplest stress we have to consider on the rim bolts is one of pure tension, due to the centrifugal force of the halves of the pulley, A safe assumption to make is that the rim is free

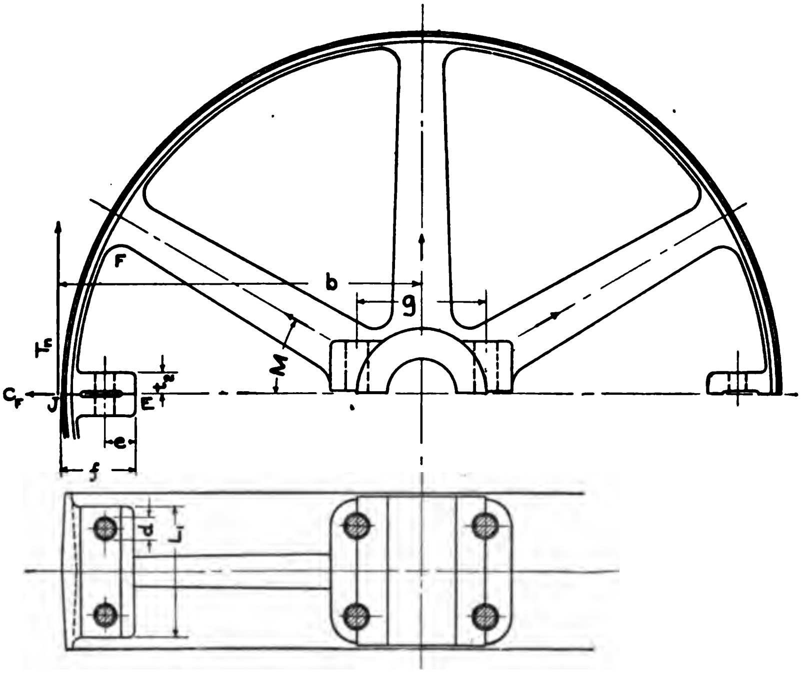


Fig. 24.

from the arms and hub, as in the simple pulley, and that the centrifugal force developed by it has to be taken by the rim bolts alone. In other words, consider the rim bolts as belonging entirely to the rim, and make them as strong as the rim, leaving the hub bolts to take the centrifugal force of the arms and hub, and the spreading tendency due to the key.

Another tensile stress is induced in the rim bolts by the fact, that, having made an open joint in the rim, and in addition placed the extra weight of lugs there, the centrifugal action at this point is increased, and at the same time a point of weakness in the rim

introduced. Referring to Fig. 24, the rim flanges EJ tend to fly out due to the centrifugal force C_F. This tends to open the joint J at the outside of the rim ; to throw a bending stress on the rim, maximum at the point F ; and to "heel" the rim flanges about the point E. The rim bolts acting on the leverage e about the point E must resist these tendencies, and are thereby put in tension.

Referring to equation 18, we find the intensity of stress due to the centrifugal force of the rim in lbs. per square inch to be :

$$p = \frac{v^2}{10}.$$

If A is the sectional area of the rim in square inches, this means that the total strength of the rim is represented by $\frac{Av^2}{10}$. The strength of a bolt is represented by the expression $\frac{S\pi d_1^{\,2}}{4}$. If, now, there are n bolts in the flange, the total resisting force of the bolts is $\frac{nS\pi d_1^{\,2}}{4}$; and the equation representing equality of strength between rim and bolts is :

$$\frac{Av^2}{10} = \frac{nS\pi d_1^{\,2}}{4}, \quad \textbf{(28)}$$

from which, by a proper assumption of the fiber stress S, which should be low, the opening-up tendency of the joint being neglected, the diameter at the root of the thread d_1 may be calculated, and the nominal bolt diameter chosen. Reference to the table for strength of bolts, given in the chapter on Bolts, Studs, etc., will be found convenient.

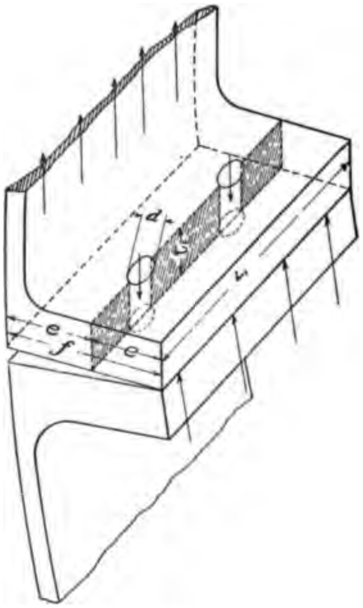


Fig. 25.

It is very doubtful if the tension on the flange bolts, due to the "heeling" about E can be calculated with sufficient accuracy to be of much value. It is probably better to assume S at a low value, say 4,000, and, in addition, for large and high-speed pulleys, stiffen the rim by running a rib between the flange and the adjacent arm. It is evident that if we make the rim so stiff that it cannot deflect, there will be no "heeling" about E; and the bolts will be well proportioned by the preceding calculation, giving them equal strength to that of the rim section.

For the bolt flange itself, any tendency to open at the joint J would cause it to act like a beam loaded at some point near its middle with the bolt load, and supported at J and E. This condition is shown in Fig. 25. Probably the weakest section would be along the line of the bolt centers. We have just noted that the carrying capacity of the bolts is $\frac{nS\pi d_1^2}{4}$. Hence, assuming that $e = \frac{1}{2}f$, which is about the worst case which could happen, we have a beam of length f loaded at the middle with $\frac{nS\pi d_1^2}{4}$ and supported at the ends. Equating the external moment to the internal moment, we have :

$$\frac{nS\pi d_1^2}{4} \times \frac{f}{4} = \frac{s(L_1 - nd)t_2^2}{6}, \qquad (29)$$

from which the fiber stress s in the flange may be calculated and judged for its allowable value.

L_1 may be assumed a little narrower than the pulley face; and t_2 from 1 inch to 2 inches or more, depending on the thickness of the rim.

The hub bolts doubtless assist the rim bolts in preventing the halves of the pulley from flying apart. They also clamp the hub tightly to the shaft, preventing any looseness on the key. Their function is a rather general one; and the specific stress which they receive is practically impossible to calculate. As a matter of fact, if the hub bolts were left out entirely, the pulley would still drive fairly well, but general rigidity and steadiness would be impaired. Hence the size of the hub bolts is more a practical question than one involving calculation. The rim bolts

should be figured first, and their size determined on; then the hub bolts can be judged in proportion to the rim bolts, the diameter of shaft, the thickness and length of the hub, and the general form of the pulley. Often appearance is the deciding factor, it being manifestly inconsistent to associate small fastenings with large shafts or hubs, even though the load be actually small.

PRACTICAL MODIFICATION. Practical considerations are chiefly responsible for the location of the joint in a split pulley between the arms instead of directly at the end of an arm, where theoretically it would seem to be required. It is usually more convenient in the foundry and machine shop to have the joint between the arms; so we generally find it placed there, and strength provided to permit this. It is possible, however, to provide a double arm, or a single split arm, in which case the joint of the pulley comes at the arm, and the "heeling" action of the rim flanges is prevented.

The rim bolts should be crowded as close as possible to the rim in order to reduce the stress on them, and also the stress in the flange itself. The practical point must not be forgotten, however that the bolts must have sufficient clearance to be put into place beneath the rim.

While it is evident that the rim bolts are most effective in taking care of the centrifugal action of the halves, yet in small split pulleys it is quite common to omit the rim bolts and to use the hub bolts for the double purpose of clamping the shaft and holding the two halves together. The pulley is cast with its rim continuous throughout the full circle, and it is machined in this form. It is then cracked in two by a well-directed blow of a cold chisel, the casting being especially arranged for this along the division line by cores so set that but a narrow fin of metal holds the two parts together. This provides sufficient strength for casting and turning, but permits the cold chisel to break the connection easily.

SPECIAL FORMS OF PULLEYS.

The plain cast-iron pulley has been used in the foregoing discussion as a basis of design. A pulley is, however, such a common commercial article, and finds such universal use, that

special forms, which can be bought in the open market, are not only cheaper but better than the plain cast-iron pulley, at least for regular line-shaft work.

Cast iron is a treacherous and uncertain material for rims of pulleys. It is not well suited to high fiber stresses; hence the range of speed permissible for pulley rims of cast iron is limited. Steel and wrought iron, having several times the tensional strength of cast iron, and being, moreover, much more nearly homogeneous in texture, are well suited for this work; one of the best pulleys on the market consists of a steel rim riveted to a cast-iron spider. Such an arrangement combines strength and lightness, without increasing complication or expense.

The all-steel pulley is a step further in this direction. Here the rim, arms, and hub are each pressed into shape by specially devised machinery, then riveted and bolted together. This pulley is strictly a manufactured article, which could not compete with the simpler forms unless built in large quantities, enabling automatic machinery to be used. Large numbers of pulleys are built in this way, and are put on the market at reasonable prices.

Wood-rim pulleys have been made for many years, and, except for their clumsy appearance, are excellent in many respects. The rim is built up of segments in much the same way as an ordinary pattern is made, the segments being so arranged that they will not shrink or twist out of shape from moisture. The hubs may be of cast iron, bolted to wooden webs, and carrying hard-wood split bushings, which may be varied in bore within certain limits so as to fit different sizes of shafting. The wooden pulley is readily and most often used in the split form, thus enabling it to be put in position easily at any point of a crowded shaft. It is often merely clamped in place, thus avoiding the use of keys or set screws, and not burring or roughening the shaft in any way.

PROBLEMS ON PULLEYS.

1. Calculate the tensile stress due to centrifugal force in the rim of a cast-iron pulley 30 inches in diameter, at 500 revolutions per minute.

2. The driving force of a belt on a 36-inch pulley is 800 lbs., and the belt wrap about 180°. Calculate proportions of el-

liptical arms to resist bending, the allowable fiber stress being 2,000.

3. A pulley 12 inches in diameter, $\frac{5}{8}$-inch web, 4-inch diameter hub, transmits 25 horse-power at a belt speed of 3,000 ft. per minute. Calculate the maximum shearing stress in the web.

4. In Fig. 24 assume the following data: $L_1 = 7$ inches; $t_2 = 1$ inch; $e = 1\frac{1}{2}$ inches; $f = 3$ inches; area of rim = 3 sq. in.; allowable tensile stress in rim 1,000 lbs. per sq. in. Calculate the diameter of the rim bolts.

5. Calculate the fiber stress in the rim bolt flange along the line of the bolts.

SHAFTS.

NOTATION—The following notation is used throughout the chapter on Shafts:

A_0 = Angular deflection (degrees).
B = Simple bending moment (inch-lbs.).
B_e = Equivalent bending moment (inch-lbs.).
c = Distance from neutral axis to outer fiber (inches).
d, d_0, d_2, d_3, d_4 = Diameters of shaft (inches).
d_1 = Internal diameter of shaft (inches).
E = Direct modulus of elasticity (a ratio).
e = Transverse deflection (inches).
G = Transverse modulus of elasticity (a ratio).
H = Horse-power (33,000 ft.-lbs. per minute).
I = Moment of inertia.
K = Distance between bearings (inches).
L = Length along shaft (inches).
L_1, L_2 = Length of bearings (inches.)
M = Distance between bearings (feet.)
N = Number of revolutions per minute.
P = Driving force of belt (lbs.).
P_1 = Load applied as stated (lbs.).
R = Radius at which load as stated acts (inches).
S = Fiber stress, tension, compression, or shearing (lbs. per sq. in.).
T = Simple twisting moment (inch-lbs.).
T_e = Equivalent twisting moment (inch-lbs.).
T_n = Tension in tight side of belt (lbs.).
T_o = Tension in loose side of belt (lbs.).
W = Load applied as stated (lbs.).

ANALYSIS. The simplest case of shaft loading is shown in Fig. 26. The equal forces W, similarly applied to the disc at the distance R from its center, tend to twist the shaft off, the tendency being equal at all points of the length L between the disc and the post, to which the shaft is rigidly fastened. The fastening to the post, of course, in this ideal case, takes the place of a resisting member of a machine. A state of pure torsion is induced in the shaft; and any element, such as *ca*, is distorted to the position *cb*, *aob* being the angular deflection for the distance L.

The case of Fig. 27 is illustrative of what occurs when a belt pulley is substituted for the simple disc. Here the twisting action is caused by the driving force of the belt, which is $T_n - T_o = P$,

acting at the radius R. Torsion and angular deflection exist in the shaft, as in Fig. 26. In addition, however, another stress of a different kind has been introduced; for not only does the shaft tend to be twisted off, but the forces T_n and T_o, acting together, tend to bend the shaft, the bending moment varying with every section of the shaft, being nothing at the point *o*, and maximum at the point *c*. This combined action is the most common of any that we find in ordinary machinery, occurring in nearly every case with which we have to deal.

In Fig. 27, if the forces T_n and T_o be made equal, there will be no tendency at all to twist off the shaft, but the bending will remain, being maximum at the point *c*. This condition is illustrative of the case of all ordinary pins and studs in machines. In this sense, a pin or a stud is simply a shaft which is fixed to the frame of the machine, there being no tendency to turning of the pin or stud itself. The same condition would be realized if the disc in Fig. 27 were loose upon the shaft. In that case, the bending moment would be caused by $T_n + T_o$ acting with the leverage L. Of course there would have to be some resistance for $T_n - T_o$ to work against, in order that torsion should not be transmitted through the shaft. This condition might be introduced by having a similar disc lock with the first one by means of lugs on its face, thus receiving and transmitting the torsion.

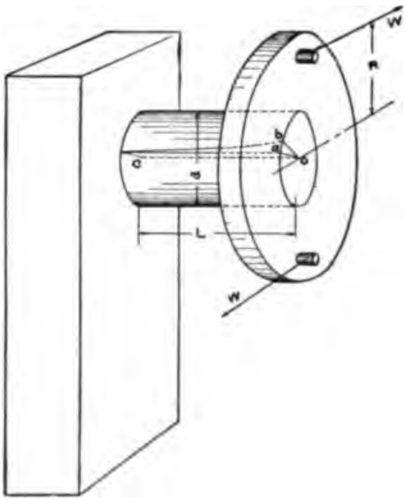


Fig. 26.

If the distance L becomes very great, both the angular deflection due to twisting, and the sidewise deflection due to bending, become excessive, and not permissible in good design. This trouble is remedied by placing a bearing at some point closer to the disc, which, as it decreases L, of course, decreases the bending moment and therefore the transverse deflection. The angular de-

flection can be decreased only by bringing the resistance and load nearer together.

The above implies, of course, that the diameter of the shaft is not changed, it being obvious that increase of diameter means increase of strength and corresponding decrease of both angular and transverse deflection.

If the speed of the shaft be very high, and the distance between bearings, represented by L, be very great, the shaft will take a shape like a bow string when it is vibrated, and smooth action cannot be maintained.

It is necessary to carry the cases of Figs. 26 and 27 but a

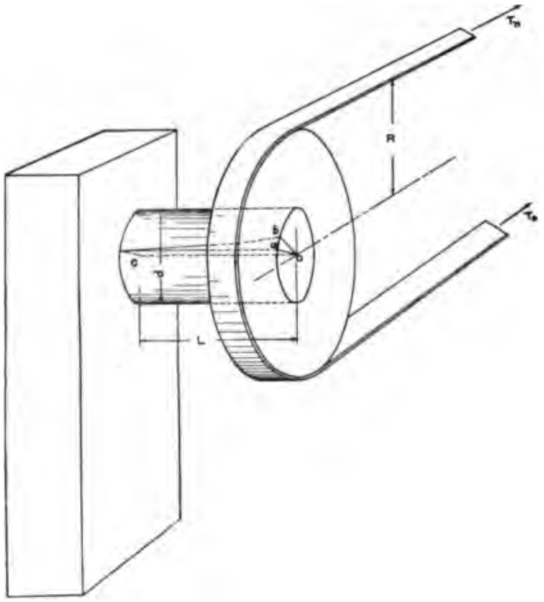


Fig. 27.

single step farther to illustrate the actual working conditions of shafting in machines. Suppose the rigid post to have the shaft passing clear through it, and to act as a bearing, so that the shaft can freely rotate in it, the resistance being exerted somewhere beyond. The twisting moment will be unchanged, also the bending moment; but the effect of the bending moment will be on each particle of the shaft in succession, now putting compression on a given particle, and then tension, then compression again, and so on, a complete cycle being performed for each revolution. This

brings out a very important difference between the bending stress in pins and the bending stress in rotating shafts. In the one case the bending stress is non-reversing; in the other, reversing; and a much higher fiber stress is permissible in the former than in the latter.

THEORY—Simple Torsion. In the case of simple torsion the stress induced in the shaft is a shearing one. The external moment acts about the axis of the shaft, or is a polar moment; hence in the expression for the moment of the internal forces, the polar moment of inertia must be used. Now, from mechanics we have:

$$T = \frac{SI}{c};$$

and $$\frac{I}{c} = \frac{d^3}{5.1} \text{ (for circular section of diameter } d\text{)};$$

therefore, $$T = \frac{Sd^3}{5.1}, \qquad (30)$$

from which the diameter for any given twisting moment and fiber stress can readily be found.

For a hollow shaft this expression becomes:

$$T = \frac{S(d_0^4 - d_1^4)}{5.1d_0}. \qquad (31)$$

Simple Bending. The stresses induced in a pin or shaft under simple bending are compression and tension. The external moment in this case is transverse, or about an axis across the shaft; hence the direct moment of inertia is applicable to the equation of forces.

$$B = \frac{SI}{c};$$

and $$\frac{I}{c} = \frac{d^3}{10.2} \text{ (for circular section of diameter } d\text{)};$$

therefore, $$B = \frac{Sd^3}{10.2}. \qquad (32)$$

For a hollow shaft or pin this expression becomes:

$$B = \frac{S(d_0^4 - d_1^4)}{10.2d_0}. \qquad (33)$$

Combined Stresses. In the greater number of cases met with

in practice, we find two or more simple stresses acting at the same time, and, although the shaft may be strong enough for any one of them alone, it may fail under their combined action. The most common cases are discussed below.

Tension or Pressure Combined with Bending. In Fig. 28, the load W produces a tension acting over the whole area of d, due to its direct pull. It also produces a bending action due to the leverage R, which puts the fibers at B in tension and those at the opposite side in compression. It is evident, therefore, that by taking the algebraic sum of the stresses at either side we shall obtain the net stress. It is also evident that the greatest and

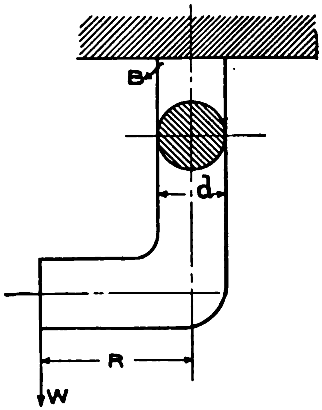


Fig. 28.

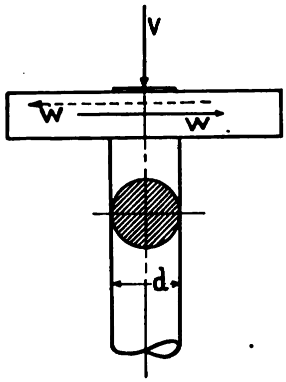


Fig. 29

controlling stress will occur on the side where the stresses add, or on the tension side. Hence, from mechanics,

$$W = \frac{\pi d^2 S}{4};$$

or, $$S = \frac{4W}{\pi d^2} \text{ (due to direct tension).} \quad (34)$$

Also, $$WR = \frac{Sd^3}{10.2};$$

or, $$S = \frac{10.2\ WR}{d^3} \text{ (due to bending).} \quad (35)$$

Hence the combined tensional stress acting at the point B, or, in

fact, at any point on the extreme outside of the vertical shaft toward the force W, is:

$$S = \frac{4W}{\pi d^2} + \frac{10.2\ WR}{d^3}. \qquad (36)$$

If W acted in the opposite direction, the greatest stress would still be at the side B, but would be a compression instead of a tension, of the same magnitude as before.

Tension or Compression Combined with Torsion. In Fig. 29, V might be the end load on a vertical shaft; and the two forces W might act in conjunction with it as in the case of Fig. 26, at the radius R. This case is not very often met with. It is usually possible to combine the moments, find an equivalent moment of a simple kind, and use the corresponding simple fiber stress. In the case in question we have a direct stress to be combined with a shearing stress, and mechanics gives us the following solution:

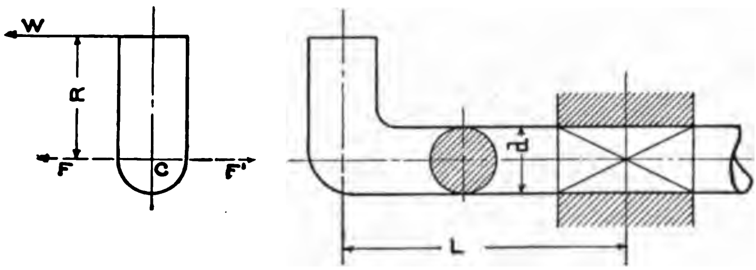


Fig. 30.

Let S_s = simple shearing stress (lbs. per sq in.).
Let S_c = simple compressive stress (lbs. per sq. in.).
Let S_{rs} = resultant shearing stress (lbs. per sq. in.).
Let S_{rc} = resultant compressive stress (lbs. per sq. in.).

We then have :

$$2WR = \frac{S_s d^3}{5.1};$$

or,

$$S_s = \frac{5.1(2WR)}{d^3}. \qquad (37)$$

Also,

$$V = \frac{\pi d^2 S_c}{4};$$

or,
$$S_c = \frac{4}{\pi d^2} V. \tag{38}$$

Now, from a solution given in simplest form in "Merriman's Mechanics"—which the student may consult, if desired—values for the resultant stresses can be found. Whichever of these is the critical one for the material used, should form the basis for its diameter:

$$S_{rs} = \sqrt{S_s^2 + \frac{S_c^2}{4}}. \tag{39}$$

Also,
$$S_{rc} = \frac{S_c}{2} + \sqrt{S_s^2 + \frac{S_c^2}{4}}. \tag{40}$$

Bending Combined with Torsion. In Fig. 30, the load W acts not only to twist the shaft off, but also presses it sidewise against the bearing. As it is usually customary to figure the maximum moment as taking place at the center of the bearing, the length L, which determines the bending moment, is taken to that point. The theory of the stress induced in this case is complicated. In order to make the magnitude of the moments clearer, let us introduce the two equal and opposite forces F and F^1, each equal to W, at the point C. We can evidently do this without changing the equilibrium of the shaft in any way. We now see that W and F^1 act as a couple giving a twisting moment WR; and that F acts with a leverage L, producing a bending moment FL = WL, at the middle of the bearing.

If, now, we find an equivalent twisting moment, or an equivalent bending moment, which would produce the same effect on the fibers of the shaft as the two combined, we can treat the calculation of the diameter as a simple case, and proceed as in the cases of simple torsion and simple bending considered above. This relation is given us in mechanics:

$$B_e = \frac{B}{2} + \frac{1}{2}\sqrt{B^2 + T^2}. \tag{41}$$

$$T_e = B + \sqrt{B^2 + T^2}. \tag{42}$$

These expressions are true in relation to each other, on the assumption that the allowable fiber stress S is the same for tension, com-

pression, and shearing. For the material of which shafts are usually made, this is near enough to the truth to give safe and practical results. Using the expressions for internal moments of resistance as previously noted for circular sections, we then have :

$$B_e = \frac{Sd^3}{10.2}. \qquad (43)$$

Also,

$$T_e = \frac{Sd^3}{5.1}. \qquad (44)$$

Either equation may be used ; the diameter d will result the same whichever equation is taken. For the sake of simplicity, equation 42 is generally preferred, equation 44 being taken in conjunction with it.

The expression $\sqrt{B^2 + T^2}$ is one that would be a long and tedious task to calculate. By inspection it is readily seen that this quantity can be graphically represented by means of a right-angled triangle having B and T as the sides. We may then lay down on a piece of paper, to some convenient scale, the moments B and T as the sides of a right-angled triangle, when, upon measuring the hypothenuse, we can easily read off to the same scale $\sqrt{B^2 + T^2}$. Even if the drawing is made to a small scale, the accuracy of the reading will be sufficient to enable the value for d to be solved very closely. This graphical method is illustrated in Part I.

Deflection. For a shaft subjected to pure torsion, as in Fig. 26, the angular deflection due to the load may be carried to a certain point before the limit of working fiber stress is exceeded. The equation worked out from mechanics for this condition, is:

$$A^\circ = \frac{584\ TL}{Gd^4}, \qquad (45)$$

which at once gives the number of degrees of angular deflection for a shaft whose modulus of elasticity, torsional moment, and length are known.

The shearing modulus of elasticity of ordinary shaft steel runs from 10,000,000 to 13,000,000, giving as an average about 12,000,000.

By the well-known relation of "Hooke's law" (stresses proportional to strains within the elastic limit of the material), we have:

$$\frac{A^\circ}{360^\circ} = \frac{SL}{\pi G d};$$

or

$$S = \frac{A\pi G d}{360\ L}. \qquad (46)$$

A twist of one degree in a length of twenty diameters is a usual allowance. Substituting A = 1, L = 20d, and G = 12,000,000, we have:

$$S = 5{,}240 \text{ (nearly)}. \qquad (47)$$

This is a safe value for shearing fiber stress in steel. In fact, in calculations for strength, even for reversing stresses, the usual figure is 8,000 (lbs. per square inch), thus indicating that the relation of one degree to twenty diameters is well within the limit of strength.

For a hollow shaft the above formula becomes :

$$A^\circ = \frac{584\ TL}{G(d_o^4 - d_1^4)}. \qquad (48)$$

Transverse deflection occurs when the shaft is subjected to a bending moment. It may therefore exist alone or in conjunction with angular deflection. Transverse deflection of shafts, however, rarely exists up to the point of limiting fiber stress, because before that point is reached the alignment of the shaft is so disturbed that it is not practicable as a device for transmitting power. A transverse deflection of .01 inch per foot of length is a common allowance ; but it is impossible to fix any general limit, as in many cases this figure, if exceeded, would do no harm, while in others—such as heavily loaded or high-speed bearings—even the figure given might be fatal to good operation.

The formula for transverse deflection, deduced from mechanics, varies with the system of loading. The three most common conditions only are given below, reference to the handbook being necessary if other conditions must be satisfied:

Fixed at one end, loaded at the other,

$$e = \frac{WL^3}{3\ EI}. \qquad (49)$$

Supported at ends, loaded in middle,

$$e = \frac{WL^3}{48EI}. \qquad (50)$$

Supported at ends, loaded uniformly,

$$e = \frac{5WL^3}{384EI}. \qquad (51)$$

For transverse deflection the direct modulus of elasticity must be used, for the fibers are stretched or compressed, instead of being subjected to a shearing action. The most usual value of the direct modulus of elasticity for ordinary steel is 30,000,000, and is denoted in most books by the symbol E. Both the shearing and direct moduli of elasticity are really nothing but the ratio of the stress to the strain produced by that stress, it being assumed that the given material is perfectly elastic. A material is supposed to be perfectly elastic up to a certain limit of stress, and it is within this limit that the relation as above holds good.

Expressed in the form of an equation this would be :

$$E = \frac{S}{\frac{e}{L}} = \frac{SL}{e}. \qquad (52)$$

Centrifugal Whirling. If a line shaft deflect but slightly, due to its own weight, or the weight or pressure of other bodies upon it, and then be run at a high speed, the centrifugal force set up increases the deflection, and the shaft whirls about the geometrical line through the centers of the bearings, causing vibration and wear in the adjoining members. It is evident that the practical remedy for this tendency in a shaft of given diameter and speed is to locate the bearings sufficiently close to render the action of small effect.

Many formulæ might be given for this relation, each being based on different assumptions. Perhaps as widely applied and as simple as any, is the "Rankine" formula, which sets the limit of length between bearings for shafts not greatly loaded by intermediate pulleys or side strains :

$$M = 175 \sqrt{\frac{d}{N}}. \qquad (53)$$

Horse-Power of Shafting. Horse-power is a certain specific rate of doing work, *viz.*, 33,000 foot-pounds per minute. Hence, to find the horse-power that a shaft will transmit, we must first find the work done, and then relate it to the speed. Take, for example, the case of a pulley, the symbols being the same as before —namely, P = driving force at rim of pulley (lbs.); R = radius of pulley (inches); N = number of revolutions per minute; and H = horse-power. Then,

$$\text{Work} = \text{force} \times \text{distance} = P \times (2\,\pi\,RN) = H \times 33{,}000 \times 12;$$

or,

$$PR = \frac{63{,}025H}{N}. \qquad (54)$$

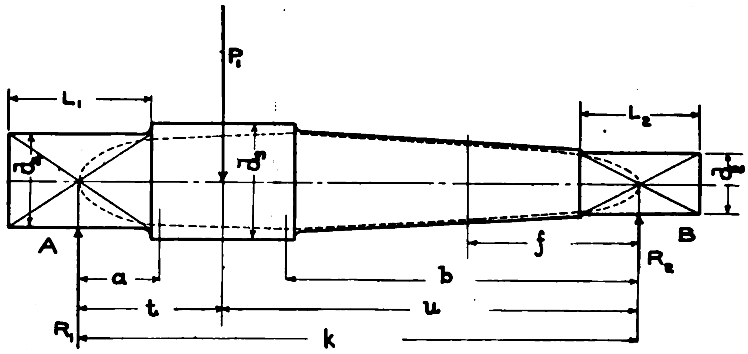


Fig. 31.

This is one of the most useful equations for calculations involving horse-power. By it the number of inch-pounds torsion for any horse-power can be at once ascertained.

It should be clearly noted, however, that in this equation the bending moment does not enter at all. Hence any shaft based in size on *horse-power alone*, is based on *torsional moment alone*, bending moment being entirely neglected. In many cases the bending moment is the controlling one as to limiting fiber stress. Hence empirical shafting formulæ depending upon the horse-power relation are unsafe, unless it is definitely known just what torsional and bending moments have been assumed.

The only safe way to figure the size of a shaft is to find accurately what torsional moment and bending moment it has to sustain, and then combine them according to equation 41 or 42

introducing the element of speed as basis for assumption of a high or low working fiber stress.

PRACTICAL MODIFICATION. The practical methods of handling the theoretical shaft equations have reference to the fit of the shaft within the several pieces upon it. The running fit of a shaft in a bearing is usually considered to be so loose that the shaft could freely deflect to the center of the bearing. This is doubtless an extreme view of the case, but it is the only safe assumption. Hence a shaft running in bearings (see Fig. 31) is supposed to be supported at the centers of those bearings, and its theoretical strength is based on this supposition.

For a tight or driving fit upon the shaft, a safe assumption to make is that there is looseness enough at the ends of the fit to permit the shaft to be stressed by the load a short distance within the faces of the hub, say from ½ inch to 1 inch. For example, referring to Fig. 31, suppose P_1 to be the transverse load, exerted through a hub fast upon the part of the shaft d_3. Taking moments about the center of one bearing, and solving for the reaction at the center of the other, we have:

$$P_1 u = R_1 K;$$

or,
$$R_1 = \frac{P_1 u}{K}. \qquad (55)$$

Also,
$$P_1 t = R_2 K;$$

or,
$$R_2 = \frac{P_1 t}{K}. \qquad (56)$$

Now, as far as the part of shaft d_3 is concerned, it may depend for its size on the bending moment $R_2 b$, or on $R_1 a$. The reason the lever arm is not taken to the point directly under the load P_1, is because it is not practically possible to break the shaft at that point, on account of the reinforcement of the hub, which is tightly fitted upon it. Trying these moments to see which is the greater, we shall find that the greater moment always occurs in connection with the longer lever arm. Hence $R_2 b$ will be greater than $R_1 a$. We then write the equation of *external moment = internal moment:*

$$R_2 b = \frac{S d_3^3}{10.2};$$

or,
$$d_3 = \sqrt[3]{\frac{10.2 \ R_2 b}{S}}. \qquad (57)$$

For the size of bearing A we have the maximum bending moment:

$$R_1 \frac{L_1}{2} = \frac{S \, d_4^3}{10.2};$$

or,
$$d_4 = \sqrt[3]{\frac{10.2 \ R_1 L_1}{2 \ S}}. \qquad (58)$$

For the size of bearing B we have the maximum moment:

$$R_2 \frac{L_2}{2} = \frac{S \, d_2^3}{10.2};$$

or,
$$d_2 = \sqrt[3]{\frac{10.2 \ R_2 L}{2 \ S}}. \qquad (59)$$

The above calculations are, of course, on the assumption that no torsion is transmitted either way through this axle. We should in that case have combined torsion and bending. This has been made sufficiently clear in preceding paragraphs and in Part I, to require no further illustration.

The dotted line in Fig. 31 shows the theoretical shape the axle should take under the assumed conditions. The practical modification of this shape is obvious. At the shoulders of the shaft the corners should not be sharp, but carefully filleted, to avoid the possible starting of a crack at those points.

Often the diameter of certain parts of a shaft may be larger than strength actually calls for. For example, in Fig. 31, the part d_3 need only be as large as the dotted line; but it is obvious that unless the key is sunk in the body of the shaft, the hub could not be slipped into place over the part d_4. If, however, the diameter d_3 be made large enough so that the bottom of the key will clear d_4, the rotary cutter which forms the key way in d_3 will also clear d_4, and the key way can be more easily produced.

In cases where fits are not required to be snug, a straight shaft of cold-rolled steel is commonly used. Here any parts fastened on the middle of the shaft have to be driven over a considerable length of the shaft before they reach their final position. Moreover, there is no definite shoulder to stop against, and measurement has to be resorted to in locating them.

It does not pay to turn any portion of a cold-rolled shaft, unless it be the very ends, for relieving the "skin tension" in such material is sure to throw the shaft out of line and necessitate subsequent straightening.

Turned-steel shafts for machines may with advantage be slightly varied in diameter wherever the fit changes; and although the production of shoulders costs something, yet it assists greatly in bringing the parts to their exact location, and enables the workman to concentrate his best skill on the fine bearing fits, and to save time by rough-turning the parts that have no fits.

Hollow shafts are practicable only for large sizes. The advantages of removing the inner core of metal, aside from some specific requirement of the machine, are that it eliminates all possibility of cracks starting from the checks that may exist at the center, permits inspection of the material of a shaft, and, in case of hollow-forged shafts, gives an opening for the forging mandrel. In the last case, the material is improved by a rolling process.

The material most common for use in machine shafting is the ordinary "Machinery Steel," made by the Bessemer process. This steel is apt to be "seamy," and often contains checks and flaws that are detected only upon sudden and unexpected breakage of a part apparently sound. This characteristic is a result of the process employed in the manufacture of the steel, and thus far has never been wholly eliminated. **Bessemer steel** is, nevertheless, a very useful material, and the above weakness is not so serious but that this kind of steel can be used with success in the great majority of cases.

When a more homogeneous shaft is desired, **open-hearth steel** is available. This is a more reliable material to use than the Bessemer, and costs somewhat more. It makes a stiff, true, fine-surfaced shaft, high-grade in every respect. It is usually specified for armature shafts of dynamos and motors.

Steels of special strength, toughness, and elasticity are made under numerous processes. **Nickel steel** is perhaps the most conspicuous example. While for this steel a high price has to be paid, yet its great strength, in connection with other valuable qualities, makes it a material extremely valuable for service where light weight is essential, or where contracted space demands small size.

The range of strength of these various steels is so great that it is well-nigh useless to go into a discussion of it here. Reference should be had to the extended discussions of the handbooks, and to special trade pamphlets. A study of the possibilities of steel in its various forms for use in shafting, is very valuable as a basis for design, as it can almost be said that a machine consists chiefly of a "collection of shafts with a structure built round them." The shafts are like a core, and evidently the size of the core determines the shell about it.

PROBLEMS ON SHAFTS.

1. Required the twisting moment on a shaft that transmits 30 horse-power at 120 revolutions per minute.
2. Find the diameter of a steel shaft designed to transmit 50 horse-power at 150 revolutions per minute.
3. Assuming same data as in Problem 1, find the diameters of a hollow shaft for a value of S = 8,000.
4. A belt on an idler pulley embraces an angle of 120 degrees. Assuming tension of belt 1,000 pounds on each side, and pulley located midway between bearings, which are 30 inches from center to center, what is the diameter of shaft required?
5. Calculate the diameter of a steel shaft designed to transmit a twisting moment of 400,000 inch-pounds and also to take a bending moment of 300,000 inch-pounds.
6. Find the angular deflection in a 4-inch shaft 20 feet long when subjected to a load of 5,500 pounds applied to an arm of 30-inch radius. Assume transverse modulus of elasticity equal to 12,000,000.
7. The overhung crank of a steam engine has a force of 32,000 lbs. at the center of the crank pin, which is 12 inches from the center of the shaft bearing, measured parallel to the shaft. The radius of crank arm is 10 inches. Assume S equal to 10,000. Calculate the diameter of the crank shaft.
8. On a short, vertical steel shaft the load is 5,000 pounds. A gear, 36 teeth, $1\frac{1}{2}$ diametral pitch, at top of shaft, transmits a load of 4,000 pounds at the pitch line. Safe shear = 7,500. What is the diameter of the shaft?

SPUR GEARS.

NOTATION—The following notation is used throughout the chapter on Spur Gears:

b = Breadth of rectangular section of arm (inches).

M. M_1 = Revolutions per minute.

μ = Coefficient of friction between teeth.

C = Width of arm extended to pitch line (inches).
c = Distance from neutral axis to outer fiber (inches).
D = Pitch diameter of gear (inches).
F = Face of gear (inches).
f = Clearance of tooth at bottom (inches).
G = Thickness of arm extended to pitch line (inches).
H = Thickness of tooth at any section (inches).
h = Depth of rectangular section of arm (inches).
I = Moment of inertia.
K = Thickness of rim (inches).
L = Distance from top of tooth to any section (inches).
N = Number of teeth.
n = Number of arms.
P = Diametral pitch (teeth per inch of diameter).
P^1 = Circular pitch (inches).
Q, Q_1 = Normal pressure between teeth (lbs.).
R, R_1 = Resultant pressure between teeth (lbs.).
r, r_1 = Radius of pitch circles (inches).
S = Fiber stress of material (lbs. per sq. in.).
s = Addendum of tooth (inches) = Dedendum of tooth.
t = Thickness of tooth at pitch line (inches).
W = Load at pitch line (lbs.).
y = Coefficient for "Lewis" formula.

ANALYSIS. If a cylinder be placed on a plane surface, with its axis parallel to the plane, an attempt to rotate the cylinder about its axis would cause it to roll on the plane.

Again, if two cylinders be provided with axial bearings, and be slightly pressed together, motion of one about its axis will cause a similar motion of the other, the two surfaces rolling one on the other at their common tangent line. If moved with care, there will be no slipping in either of the above cases—which is explained by the fact that no matter how smooth the surfaces may appear to be, there is still sufficient roughness to make the little irregularities interlock and act like minute teeth.

The magnitude of the force possible to be transmitted depends not only on the roughness of the surfaces, but on the amount of pressure between them. Suppose that one cylinder is a part of a hoisting drum, on which is wound a rope with a weight attached. We can readily make the weight so great that, no matter how hard we press the two cylinders together, the driving cylinder will not turn the hoisting cylinder, but will slip past it. If now, instead of increasing the pressure, which is detrimental both to cylinders and bearings of same, we increase the coarseness of the surfaces, or, in other words, put teeth of appreciable size on these surfaces, we attain the desired result of positively driving without excessive side pressure.

These artificial projections, or teeth, must fit into one another; hence the surfaces of the original cylinders, having been broken up into alternate projections and hollows, have entirely disap-

peared to the eye; they nevertheless exist as ideal or imaginary surfaces, which roll together with the same surface velocities as if in bodily form, provided that the curves of the teeth are correctly formed. Several mathematical curves are available for use as tooth outlines, but in practice the **involute** and **cycloidal curves** are the only ones used for this purpose.

The ideal surfaces are known as **pitch cylinders** or **pitch circles.** In Fig. 32 is shown an end view of such a pair of cylinders in contact at their pitch point P. In gear calculations we assume that there is no slip between the pitch circles, acting as driving cylinders; hence the speeds of the two pitch circles at the

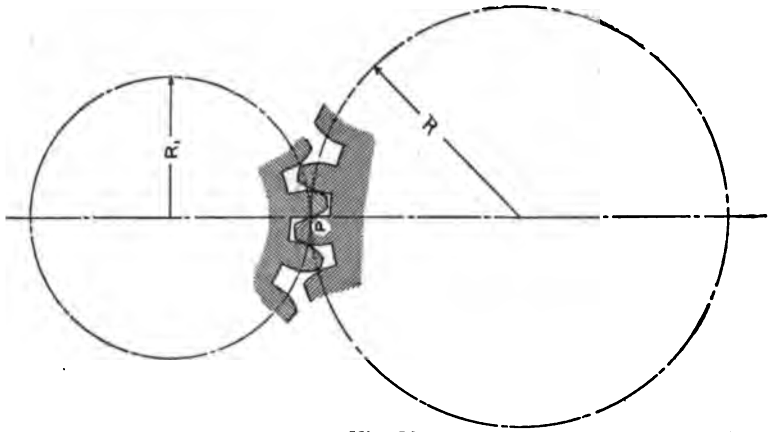


Fig. 32.

pitch point are equal. If M and M_1 be the revolutions per minute of the cylinders respectively, r and r_1 their radii, then

$$2 \pi r M = 2 \pi r_1 M_1;$$

or,
$$\frac{M}{M_1} = \frac{r_1}{r}. \qquad (60)$$

That is, the number of revolutions varies inversely as the radii.

The simple calculation as above is the key to all calculations involving gear trains in reference to their speed ratio.

Fig. 33 represents cycloidal teeth in the two extreme positions of beginning and ending contact. The normal pressure Q or Q_1 between the teeth in each position acts through the pitch point O, as it must always do in order to insure the condition of ideal roll-

ing of the pitch circles, and the velocity ratio proportional to $\frac{r_1}{r}$ As the surfaces of the teeth slide together, frictional resistance is produced at their point of contact. This force is widely variable, depending on the material and condition of the tooth surfaces, whether smooth and well lubricated, or rough and gritty. As this resistance acts in conjunction with the normal force between the teeth, we may construct a parallelogram of forces on these two as a base, the resultant pressure between the teeth being slightly changed thereby, as shown in Fig 33.

Assuming a coefficient of friction μ, the force of friction is μ Q or μ Q_1 and the resultant pressure R or R_1.

Tooth B of the FOLLOWER is therefore under a heavy bending moment measured by the product RL, L being the perpendicular distance from the center of the tooth at its base to the line of the force. This tooth also has a relatively small compressive stress due to the resolved part of R along the radius, and a relatively small shearing stress due to the resolved part of R along a tangent to the pitch circle.

Tooth D of the driven wheel or FOLLOWER has a relatively large shearing stress, a small bending moment, and practically no direct compressive stress.

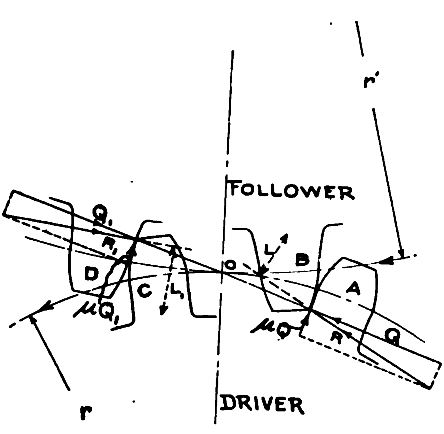


Fig. 33.

Tooth A of the driving wheel or DRIVER has a relatively large shearing stress, a small bending moment, and small compressive stress.

Tooth C of the DRIVER has a large bending moment, but small compressive and shearing stresses.

The conditions as noted above are not those of every pair of gears, in fact they vary with every difference of pitch circle, or of detail and position of tooth. It is true, however, that in nearly all cases in practice the bending stress is the controlling one from a theoretical standpoint. Moreover, the designer must consider the form and strength of the tooth when it is under the condition of maximum moment. This evidently, from the above, occurs at the beginning of contact, for the follower teeth; and at the end of contact, for the driver teeth. In the particular case illustrated in

Fig. 33, if the material in both gears were the same, tooth C, being the weaker at the root, would probably break before B; but if C were of steel, and B of cast iron, B might break first.

It will be noticed that R is nearly parallel to the top of the tooth; and it may easily happen that the friction may become of such a value that it wili turn the direction of R until it lies along the top of the tooth exactly, which is the condition for maximum moment. For strength calculations it is usual to consider this condition as existing in all cases.

At the beginning of contact there is more or less shock when the teeth strike together, and this effect is much more evident at high speeds. There is also at the beginning of contact a sort of chattering action as the driving tooth rubs along the driven tooth.

Uniform distribution of pressure along the face of the tooth is often impaired by uneven wear of the bearings supporting the gear shafts, the pressure being localized on one corner of the tooth. The same effect is caused by the accidental presence of foreign material between the teeth. Again, in cast gearing, the spacing may be irregular, or, on account of draft on the pattern, the teeth may bear at the high points only. While it is usual to consider that the load is evenly distributed along the face of the tooth, yet the above considerations show that *an ample margin of strength must always be allowed* on account of these uncertainties.

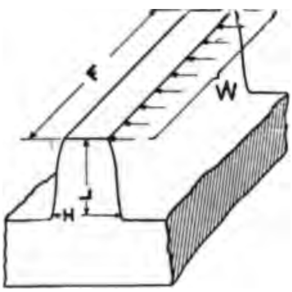


Fig. 34.

When the number of teeth in the mating gears is high, the load will be distributed between several teeth; but, as it is almost certain that at some time the proper distribution of load will not exist, and that one tooth will receive the full load, it is considered that practically the only safe method is so to design the teeth that a single tooth may be relied upon to withstand the full load without failure.

THEORY. Based on the Analysis as given, the theory of gear teeth assumes that one tooth takes the whole load, and that this load is evenly distributed along the top of the tooth and acts parallel with

its base, thus reducing the condition of the tooth to that of a cantilever beam. The magnitude of this load at the top of the tooth is taken for convenience the same as the force transmitted at the pitch circle. This condition is shown in Fig. 34. Equating the external moment to the internal moment, we then have, from mechanics:

$$WL = \frac{SI}{c} = \frac{SFH^2}{6}. \qquad (61)$$

The thickness H is usually taken either at the pitch line or at the root of the tooth just before the fillet begins; and L, of course, is dependent on the tooth dimensions. The formula is most readily used when the outline of the tooth is either assumed or known, a trial calculation being made to see if it will stand the load, and a series of subsequent calculations followed out in the same way until a suitable tooth is found. This method is pursued because there are certain even pitches which it is desirable to use; and it is safe tc say that any calculation figured the reverse way would result in fractional pitches. The latter course may be used, however, and the nearest even pitch chosen as the proper one.

As stated under "Analysis," there are a great many circumstances attending the operation of gears which make impossible the purely theoretical application of the beam formulæ. For this reason there is no one element of machinery which depends so much on experience and judgment for correct proportion as the tooth of a gear. Hence it is true that a rational formula based on the theoretical one is really of the greater practical value in tooth design.

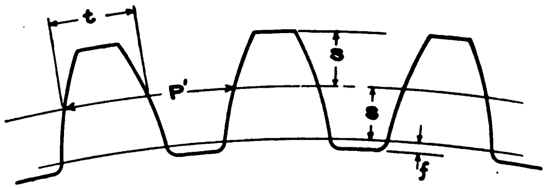
If we examine formula 61, we find that in a form solved for W, we have:

$$W = \frac{SFH^2}{6\,L}. \qquad (62)$$

Of these quantities, H and L are the only variables, for we can make the others what we choose. H and L depend upon the circular pitch P^1 and the curvature and outline of the tooth. If now we could settle on a standard system of teeth, we could establish a coefficient to be used to take the place of the variable part

of H and L, which depends on the outline of tooth, and we should thus have an empirical formula which would be on a theoretical basis.

This, Mr. Wilfred Lewis has done; and it is safe to say that this formula is more universally used and with more satis-



$t = \frac{P^1}{2}$

$f = \frac{t}{10}$

$s = \frac{P^1}{\pi}$

Fig. 35.

factory practical results than any other formula, theoretical or practical, that has ever been devised. His coefficient is known as y, and was determined from many actual drawings of different forms of teeth showing the weakest section. This coefficient is worked out for the three most common systems as follows:

For 20° involute, $y = 0.154 - \frac{0.912}{N}$. **(63)**

For 15° involute and cycloidal, $y = 0.124 - \frac{0.684}{N}$. **(64)**

For radial flanks, $y = 0.075 - \frac{0.276}{N}$. **(65)**

The tooth upon which the above is based is the American standard or Brown & Sharpe tooth, for which the proportions are shown in Fig. 35.

The "Lewis" formula* is:

$W = SP^1 Fy.$ **(66)**

A table indicating the value of S for different speeds follows:

Safe Working Stresses for Different Speeds.

Speed of teeth, ft. per min.	100	200	300	600	900	1200	1800	2400
Cast iron	8000	6000	4800	4000	3000	2400	2000	1700
Steel	20000	15000	12000	10000	7500	6000	5000	4300

*NOTE. A full and convenient statement of the Lewis formula will be found in "Kent's Pocket Book."

A usual relation of F to P^1 is:

$$\text{For cast teeth, } F = 2P^1 \text{ to } 3P^1. \qquad (66)$$

$$\text{For cut teeth, } F = 3P^1 \text{ to } 4P^1. \qquad (67)$$

The usual method of handling these formulæ is as follows:

The pitch circles of the proposed gears are known or can be assumed; hence W can readily be figured, also the speed of the teeth, whence S can be read from the table. The desired relation of F to P^1 can be arbitrarily chosen, when P^1 and y become the only unknown quantities in the equation. A shrewd guess can be made for the number of teeth, and y calculated therefrom. Then solve the equation for P^1 which will undoubtedly be fractional. Choose the nearest even pitch, or, if it is desired to keep an even diametral pitch, the fractional pitch that will bring an even diametral pitch. Now, from this final and corrected pitch, and the diameter of the pitch circle, calculate the number of teeth N in the gear. Check the assumed value of y by this positive value of N.

Another good way of using this formula is to start with the pitch and face desired, and the diameter of the pitch circle. In

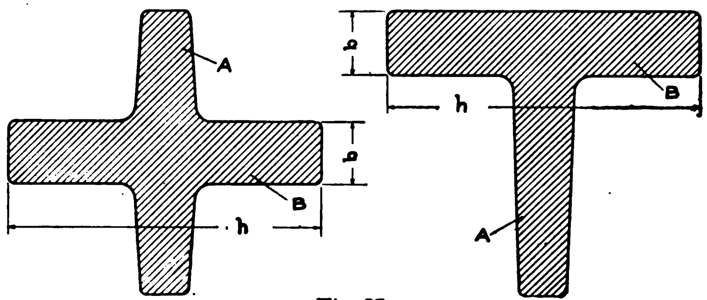


Fig. 37.

this case W is the only unknown quantity, and when found can be compared with the load required to be carried. If too small, make another and successive calculations until the result approximates the required load.

SPUR GEAR RIM, ARMS, AND HUB.

ANALYSIS and THEORY. The rim of a gear has to transmit the load on the teeth to the arms. It is thus in tension on one side of the teeth in action, and in compression on the other. The section of the rim, however, is so dependent on other practical considerations which call for an excess of strength in this respect, that

it is not considered worth while to attempt a calculation on this basis.

Gears seldom run fast enough to make necessary a calculation for centrifugal force; and in general it can be said that the design of the rim is entirely dependent on practical considerations. These will appear later under "Practical Modification."

The arms of a gear are stressed the same as pulley arms, the same theory answering for both, except that a gear rim always being much heavier than a pulley rim, the distribution of load amongst the arms is better in the case of a gear than of a pulley, and it is usually safe to assume that each arm of a gear takes its full proportion of load; or, for an oval section, equating the external moment to the internal moment as in the case of pulleys, we have:

$$\frac{WD}{n2} = 0.0393\ Sh^3. \tag{68}$$

Heavy spur gears have the arms of a cross or T section (Fig.

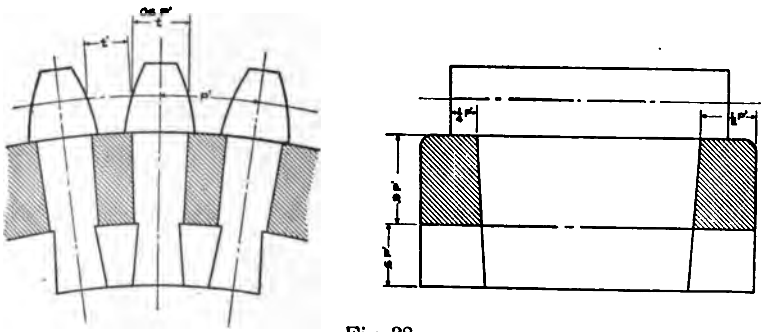


Fig. 38.

37), the latter being especially applicable to the case of bevel gears where there is considerable side thrust. The simplest way of treating such sections is to consider that the whole bending moment is taken by the rectangular section whose greater dimension is in the direction of the load. The rest of the section, being close to the neutral axis of the section, is of little value in resisting the direct load, its function being to give sidewise stiffness. The equation for the cross or T style of arm, then is:

$$\frac{W}{n} \times \frac{D}{2} = \frac{Sbh^2}{6}. \tag{69}$$

Either b or h may be assumed, and the other determined. As a guide to the section, b may be taken at about the thickness of the tooth.

Gear hubs are in no wise different from the hubs of pulleys or other rotating pieces. The depth necessary for providing sufficient strength over the key to avoid splitting is the guiding element, and can usually be best determined by careful judgment.

PRACTICAL MODIFICATION. The practical requirements, which no theory will satisfy, are many and varied. Sudden and severe shock, excessive wear due to an atmosphere of grit and corrosive elements, abrupt reversal of the mechanism, the throwing-in of clutches and pawls, the action of brakes—these and many other influences have an important bearing on gear design, but not one that can be calculated. The only method of procedure in such cases is to base the design on analysis and theory as previously given, and then add to the face of gear, thickness of tooth, or pitch an amount which judgment and experience dictate as sufficient.

Excessive noise and vibration are difficult to prevent at high speeds. At 1,000 feet per minute, gears are apt to run with an unpleasant amount of noise. At speeds beyond this, it is often necessary to provide **mortise teeth,** or teeth of hard wood set into a cast-iron rim (see Fig. 38). Rawhide pinions are useful in this regard. Fine pitches with a long face of tooth run much more smoothly at high speeds than a coarse pitch and narrow-faced tooth of equal strength. Greater care in alignment of shafts, however, is necessary, also stiffer supports.

Should it be impracticable to use a standard tooth of sufficient strength, there are several ways in which we can increase the carrying capacity without increasing the pitch. These are:

1. Use a stronger material, such as steel.
2. Shroud the teeth.
3. Use a hook tooth.
4. Use a stub tooth.

Shrouding a tooth consists in connecting the ends of the teeth with a rim of metal. When this rim is extended to the top of the tooth, the process is called "full-shrouding" (Fig. 39); and when carried only to the pitch line, it is termed "half-shrouding" (Fig. 40). The theoretical effect of shrouding is to make the tooth

act like a short beam built in at the sides; and the tooth will practically have to be sheared out in order to fail. This modification of gear design requires the teeth to be cast, as the cutter cannot pass through the shrouding. The strength of the shrouded gear is estimated to be from 25 to 50 per cent above that of the plain-tooth type.

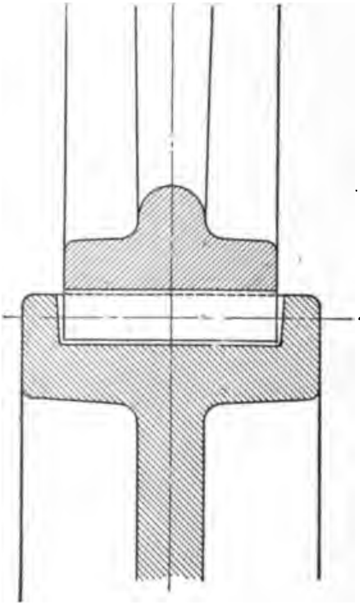


Fig. 39.

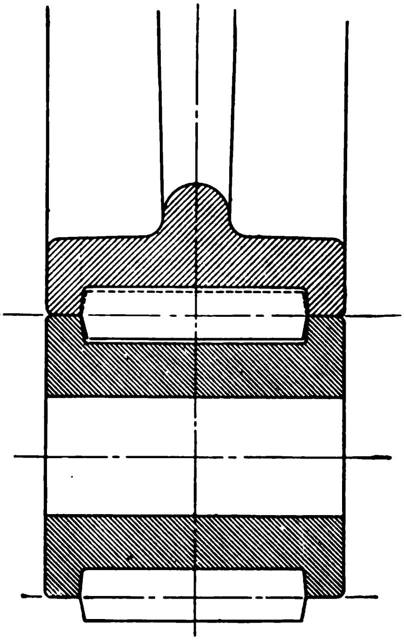


Fig. 40.

The **hook-tooth gear** (Fig. 41) is applicable only to cases where the load on the tooth does not reverse. The working side of the tooth is made of the usual standard curve, while the back is made of a curve of greater obliquity, resulting in a considerable increase of thickness at the root of the tooth. A comparison of strength between this form and the standard may be made by drawing the two teeth for a given pitch, measuring their thickness just at top of the fillet, and finding the relation of the squares of these dimensions. The truth of this relation is readily seen from an inspection of formula 61.

The **stub tooth** merely involves the shortening of the height

of the tooth in order to reduce the lever arm on which the load acts, thus reducing the moment, and thereby permitting a greater load to be carried for the same stress.

The rim of a gear is dependent for its proportions chiefly on questions of practical moulding and machining. It must bear a certain relation to the teeth and arms, so that, when it is cooling in the mould, serious shrinkage stresses will not be set up, forming pockets and cracks. Moreover, when under pressure of the cutter in the producing of the teeth, it must not chatter or spring. This condition is quite well attained in ordinary gears when the thickness of the rim below the base of the tooth is made about the same as the thickness of the tooth.

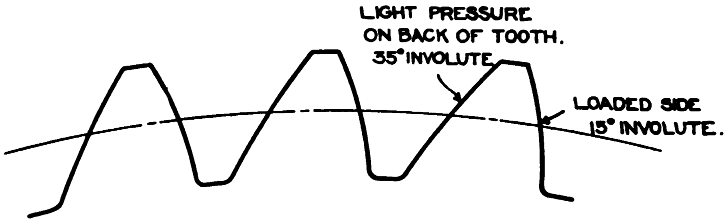


Fig. 41.

The stiffening ribs and arms must all be joined to the rim by ample fillets, and the cross-section must be as uniform as possible, to prevent unequal cooling and consequent pulling-away of the arms from the rim or hub. Often the calculated size of the arms at both rim and hub has to be modified considerably to meet this requirement.

The arms are usually tapered to suit the designer's eye, a small gear requiring more taper per foot than a large one. Both rim and hub should be tapered $\frac{1}{2}$ inch per foot to permit easy drawing-out from the mould.

The proportions given in the following table have been used with success as a basis of gear design in manufacturing practice. The table will serve as an excellent guide in laying out, and can be closely followed, in most cases with but slight modification. **Web gears** are introduced for small diameters where the arms begin to look awkward and clumsy.

Gear Design Data.

Measurements given in inches. Letters refer to Fig. 42.

Diametral pitch ..	P	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6	8
Face............	F	$6\frac{1}{4}$	$5\frac{1}{2}$	$4\frac{3}{4}$	$3\frac{3}{4}$	$3\frac{1}{4}$	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$1\frac{7}{8}$	$1\frac{1}{2}$
Thickness of arm when extended to pitch line....	G	$1\frac{3}{8}$	$1\frac{1}{4}$	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{13}{16}$	$\frac{3}{4}$	$\frac{11}{16}$	$\frac{5}{8}$	$\frac{1}{2}$
Width of arm when extended to pitch line	C	4	$3\frac{1}{2}$	3	$2\frac{1}{2}$	$2\frac{1}{4}$	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{8}$
Thickness of rim..	K	$2\frac{3}{4}$	$2\frac{3}{8}$	$2\frac{1}{8}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{3}{8}$	$1\frac{1}{4}$	1	$\frac{7}{8}$	$\frac{3}{4}$
Depth of rib......	E	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$
Thickness of web.	T	$1\frac{1}{8}$	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{3}{8}$	$\frac{5}{16}$

Number of arms, 6.
Give inside of rims and hub a draft of $\frac{1}{2}$ inch per foot.

BEVEL GEARS.

NOTATION—The following notation is used throughout the chapter on Bevel Gears:

A =Apex distance at pitch element of cone (inches).
A^1=Apex distance at bottom element of tooth (inches).
B =Angle of bottom of tooth (degrees).
C =Pitch angle (degrees).
D =Pitch diameter (inches).
E =Radius increment of gear (inches).
F =Face of gear (inches).
f =Clearance at bottom (inches).
G =Angle of face (degrees).
H =Cutting angle (degrees).
K =Radius increment of pinion (inches).
N =Number of teeth.
N^1=Formative number of teeth, or the number corresponding to the spur gear on which the outline of tooth is made.
O D=Outside diameter (inches).
P =Diametral pitch related to pitch diameter (teeth per inch).
P^1 =Circular pitch measured on the circumference of D (inches).
S =Working strength of material (lbs. per sq. in.).
s =Addendum, or height of tooth above pitch line (inches).
$s+f$=Depth of tooth below pitch line (inches).
T =Angle of top of tooth (degrees).
t =Thickness of tooth at pitch line (inches).
W =Working load at pitch line (lbs.).
y =Factor in "Lewis" formula.

ANALYSIS. It is possible to consider bevel gears as the general case of which spur gears are a special form. The pitch

surfaces of spur gears described above as cylinders, mathematically considered, are cones whose vertices are infinitely distant, while bevel gears likewise are based on pitch cones, but with a vertex at some finite point, common to the mating pair. Hence, as we might expect, the laws of tooth action are similar in bevel gears to those in the case of spur gears. The profile of the tooth in the former case, however, is based, not on the real radius of the pitch cone, but on the radius of the normal cone; and in the development of the outline the latter is treated just as though it were the radius of a spur gear. The tooth thus formed is wrapped back upon the normal cone face, and becomes the large end of the tapering bevel-gear tooth (see Fig. 44).

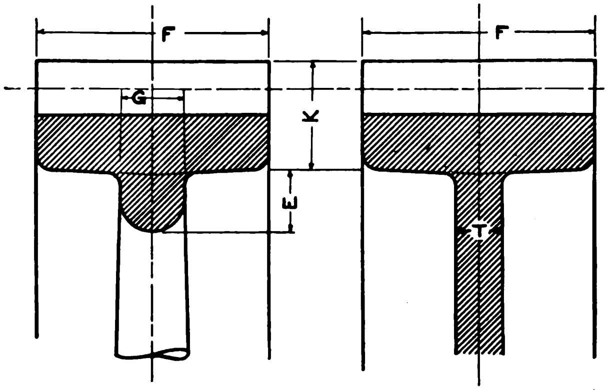


Fig. 42.

The teeth of bevel gears, being simply projections with bases on the pitch cones, have a varying cross-section decreasing toward the vertex; also a trapezoidal section of root, the latter section acting as a beam section to resist the cantilever moment due to the tooth load.

The arms must, as in the case of spur gears, transmit the load from the tooth to the shaft; in addition, the arms of a bevel gear are subjected to a side thrust due to the wedging action of the cones. Hence sidewise stiffness of the arms is more essential in this type of gear than in the case of the spur gear.

THEORY. It is evident that the calculation of tooth strength based on a trapezoidal section of root would be somewhat compli-

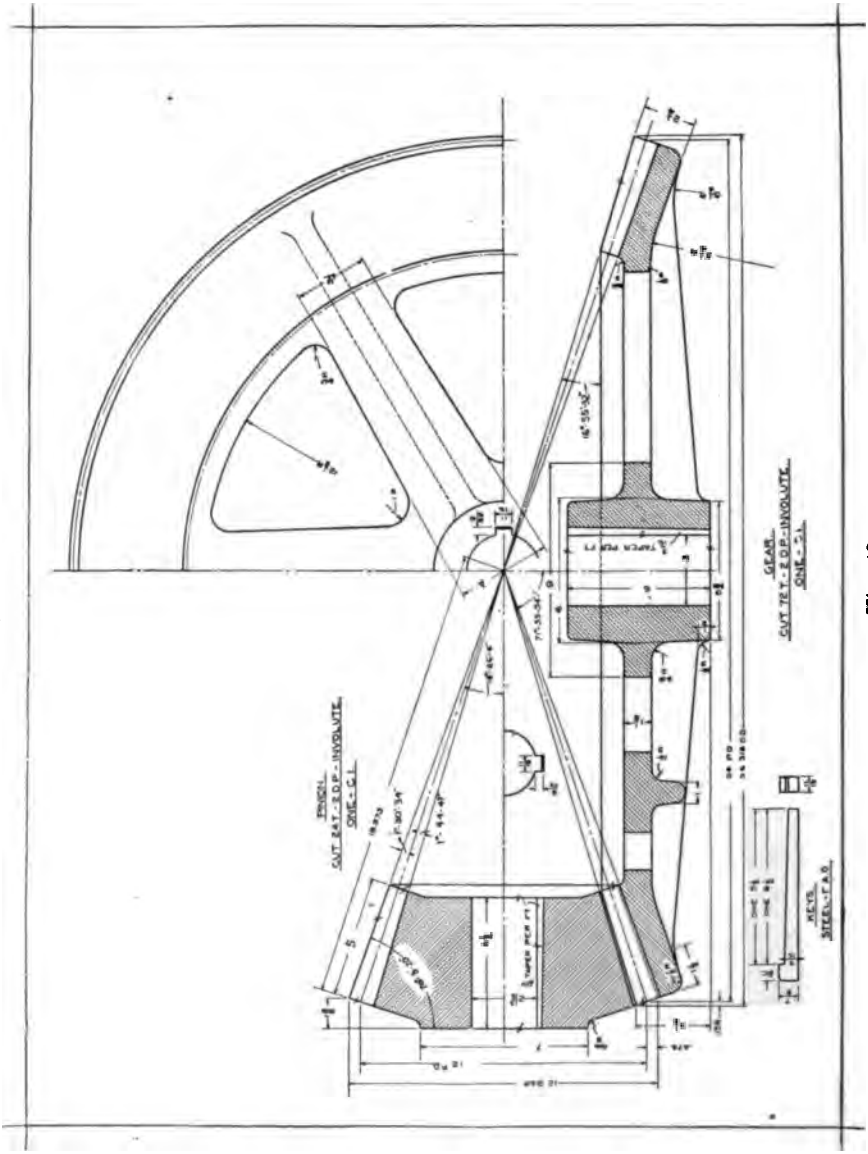


Fig. 43.

cated ; also that the trapezoid in most cases would be but little different from a true rectangle. Hence the error will be but slight if the average cross-section of the tooth be taken to represent its strength, and the calculation made accordingly.

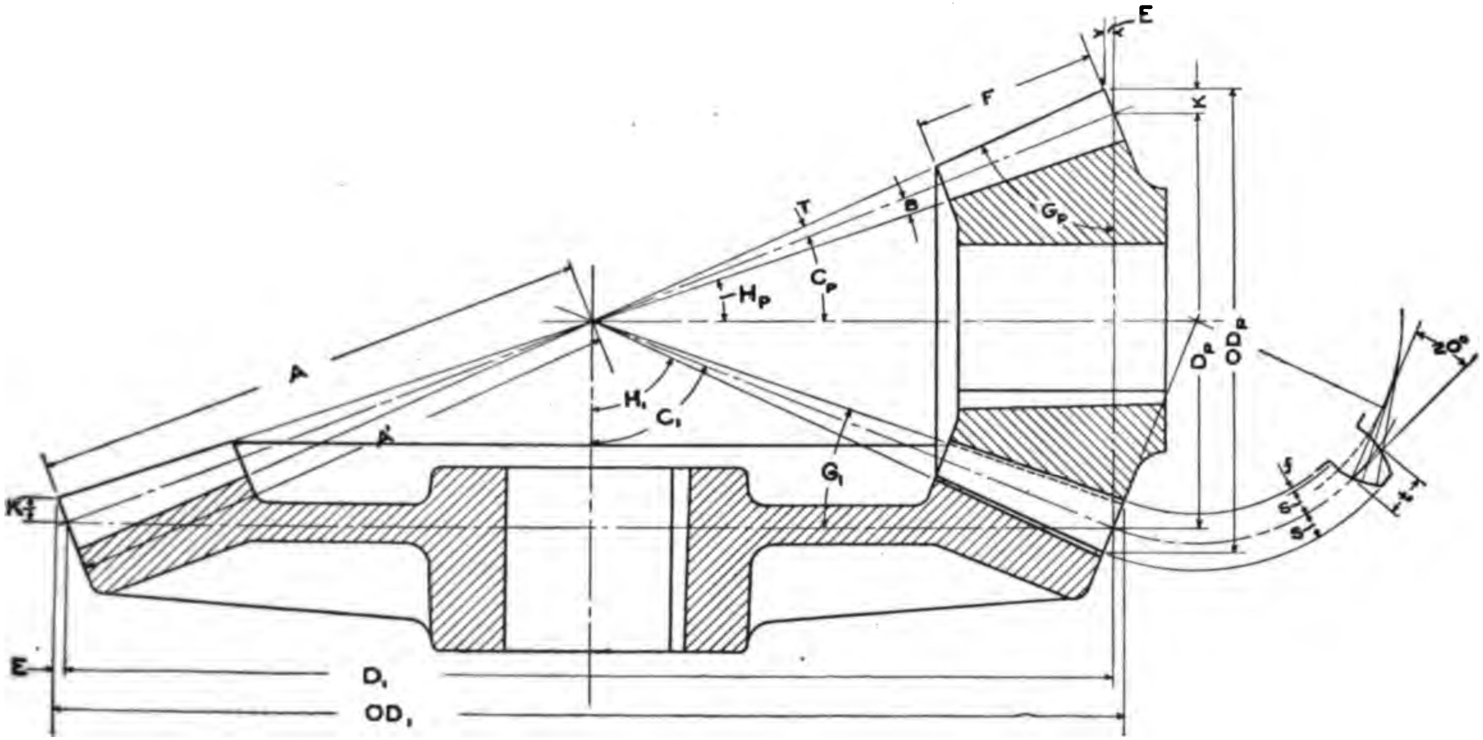


Fig. 44.

Fig. 45 shows a bevel-gear tooth with the average cross-section in dotted lines. For the purpose of calculation, the assumption is made that the section A is carried the full length of the face of the gear, and that the load which this average tooth must carry is the calculated load at the pitch line of section A. This is equivalent to saying that the strength of a bevel-gear tooth is equal to that of a spur-gear tooth which has the same face, and a section identical with that cut out by a plane at the middle of the bevel tooth. The load, as in the case of the spur gear, should be taken at the top of the tooth; and its magnitude can be conveniently calculated at the mean pitch radius of the bevel face, without appreciable error.

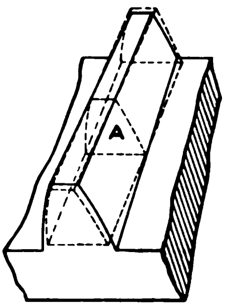


Fig. 45.

This similarity to spur gears being borne in mind, the calculation for strength needs no further treatment. Once the average tooth is assumed or found by layout, a strict following-out of the methods pursued for spur-gear teeth will bring consistent results.

The detail design of a pair of bevel gears involves some trigonometrical computations in order properly to dimension the drawing for use in finishing the blanks and subsequently in cutting the teeth, or, in the case of cast gears, in making the pattern. These calculations, although simple, are yet apt to be tedious; and inaccuracies are likely to creep in if a definite system of relations be not maintained. Hence the results of these calculations are given below in condensed and reduced form. The deduction of these formulæ is a simple and interesting exercise in trigonometry; and it is urged that they be worked out by the student from the figure, in which case he will feel greater confidence in their use.

Axes of Gears at 90 Degrees.

Use subscript 1 for gear; P for pinion. Letters refer to Fig. 44.

$$P = \frac{N}{D} = \frac{\pi}{P^1}. \tag{70}$$

$$s = \frac{1}{P} = \frac{P^1}{\pi}. \tag{71}$$

$$t = \frac{P^1}{2} = \frac{\pi}{2P}. \tag{72}$$

$$f = \frac{t}{10} = \frac{P^1}{20} = \frac{\pi}{20P}. \tag{73}$$

$$\tan C_p = \frac{N_p}{N_1};\ \tan C_1 = \frac{N_1}{N_p}. \tag{74}$$

$$\tan T = \frac{s}{A} = \frac{2 \sin C}{N}. \tag{75}$$

$$\tan B = \frac{s+f}{A} = \frac{2.314 \sin C}{N}. \tag{76}$$

$$s + f = A \tan B = \frac{1.157}{P} = 0.368P^1. \tag{77}$$

$$A = \frac{N}{2P \sin C} = \frac{1}{2P} \sqrt{N_1^2 + N_p^2} = \frac{1}{2} \sqrt{D_1^2 + D_p^2}. \tag{78}$$

$$A^1 = \frac{A}{\cos B} = \frac{N}{2P \cos B \sin C}. \tag{79}$$

$$G_1 = 90^\circ - (C_1 + T);\ G_p = 90^\circ - (C_p + T). \tag{80}$$

$$E = S \cos C_1 = S \sin C_p. \tag{81}$$

$$K = S \cos C_p = S \sin C_1. \tag{82}$$

PRACTICAL MODIFICATION. The practical requirements to be met in transmission of power by bevel gears are the same as for spur gears; but in the case of bevel gears even greater care is necessary to provide stiffness, strength, true alignment, and rigid supports. As far as the gears themselves are concerned, a long face is desirable; but it is much more difficult to gain the advantage of its strength than in the case of spur gears, because full bearing along the length of the tooth is hard to guarantee.

The rim usually requires a series of ribs running to the hub to give required stiffness and strength against the side thrust which is always present in a pair of bevel gears. Instead of arms, the tendency of bevel-gear design, except for very large gears, is toward a web on account of the better and more uniform connection thereby secured between rim and hub. This web may be lightened by a number of holes, so that the resultant effect is that of a number of wide and flat arms.

The hubs naturally have to be fully as long as those of spur gears, because there is greater tendency to rock on the shaft, due to the side thrust from the teeth, mentioned above.

The teeth on small gears are cut with rotary cutters, at least two finishing cuts being necessary, one for each side of the tapering tooth. The more accurate method is to plane the teeth on a special gear planer, and this method is followed on all gears of any considerable size. The practical requirement here is that no portion of the hub shall project so as to interfere with the stroke

of the planer tool. The requirements of gear planers vary somewhat in this regard.

Finally, after all that is possible has been done in the design of the gear itself to render it suitable to withstand the varied stresses, especial attention must be paid to the rigidity of the supporting shafts and bearings. Bearings should always be close up to the hubs of the gears, and, if possible the bearing for both pinion and gear should be cast in the same piece. If this is not done, the tendency of the separate bearings to get out of line and destroy the full bearing of the teeth is difficult to control. Thrust washers are desirable against the hubs of both pinion and gear; also proper means of well lubricating the same.

With these considerations carefully met, bevel gears are not the bugbear of machine design that they are sometimes claimed to be. The common reason why bevel gears cut and fail to work smoothly, is that the gears and supports are not designed carefully enough in relation to each other. This is also true of spur gears, but the bevel gear will reveal imperfections in its design far the more quickly of the two.

WORM AND WORM GEAR.

NOTATION—The following notation is used throughout the chapter on Worm and Worm Gear:

- D =Pitch diameter of gear (inches).
- E =Efficiency between worm shaft and gear shaft (per cent).
- f =Clearance of tooth at bottom (inches).
- i =Index of worm thread (1 for single, 2 for double, etc.).
- L =Lead of worm thread (inches).
- M =Revolutions of gear shaft per minute.
- M_w=Revolutions of worm shaft per minute.
- N =Number of teeth in gear.
- P_1 =Circular pitch = Pitch of worm thread (inches).
- R =Radius of pitch circle of worm gear (inches).
- s =Addendum of tooth (inches).
- T =Twisting moment on gear shaft (inch-lbs.).
- T_w=Twisting moment on worm shaft (inch-lbs.).
- t =Thickness of tooth at pitch line (inches).
- W =Load at pitch line (lbs.).

ANALYSIS. The simplest way of analyzing the case of the worm and worm gear is to base it upon an ordinary screw and nut. Take, for example, the lead screw of a common lathe. The carriage carries a nut, through which the lead screw passes. By the rotation of the screw, the carriage, being constrained by the guides to travel lengthwise of the ways, is moved. This motion

is, for a single-threaded screw, a distance per revolution equal to the lead of the screw.

Now, suppose that the carriage, instead of sliding along the ways, is compelled to turn about an axis at some point below the ways. Also, suppose the top of the nut to be cut off, and its length made endless by wrapping it around a circle struck from the center about which the carriage rotates. This reduces the nut to a peculiar kind of spur gear, the partial threads of the nut now having the appearance of twisted teeth.

This special form of spur gear, based on the idea of a threaded nut, is known as **a worm gear,** and the screw is termed a **worm.** The teeth are loaded similarly to those of a spur gear, but with the additional feature of a large amount of sliding along the tooth surfaces. This, of course, means considerable friction; and it is in fact possible to utilize the worm and worm gear as an efficient device, only by running the teeth constantly in a bath of oil. Even then the pressures have to be kept well down to insure the required term of life of the tooth surfaces.

It is evident that for one revolution of a single-threaded worm, one tooth of the gear will be passed. The speed ratio between the worm gear and worm shaft will then be equal to the number of teeth in the gear, which is relatively great. Hence the worm and worm gear are principally useful in giving large speed reduction in a small amount of space.

THEORY. The theory of worm-wheel teeth is complicated and obscure. The production of the teeth is simple, a dummy worm with cutting edges, called a "hob," being allowed to carve its way into the worm-gear blank, thus producing the teeth and at the same time driving the worm gear about its axis.

It is clear that if we know the torsional moment on the worm-gear shaft, and the pitch radius of the worm gear, we can find the load on the teeth at the pitch line by dividing the former by the latter. Expressed as an equation:

$$WR = T; \text{ or } W = \frac{T}{R}. \qquad (83)$$

How we shall consider this value of W as distributed on the teeth, is a question difficult to answer. The teeth not only are

curved to embrace the worm, but are twisted across the face of the gear, so that it would be practically impossible to devise a purely theoretical method of exact calculation. The most reasonable thing to do is to assume the teeth as being equally as strong as spur-gear teeth of the same circular pitch, and to figure them accordingly. It is probably true, however, that the load is carried by more than one tooth, especially in a hobbed wheel; so we shall be safe in assuming that two—and, in case of large wheels, three—teeth divide the load between them. With these considerations borne in mind, the case reduces itself to that of a simple spur-gear tooth calculation, which has already been explained under the heading "Spur Gears."

The worm teeth, or threads, are probably always stronger than the worm-gear teeth; so no calculation for their strength need be made.

The twisting moment on the worm shaft is not determined so directly as in the case of spur gears. The relative number of revolutions of the two shafts depends upon the "lead" of the worm thread and the number of teeth in the gear.

Lead (L) is the distance parallel to the axis of the worm which any point in the thread advances in one revolution of the worm. **Pitch** (P^1) is the distance parallel to the axis of the worm between corresponding points on adjacent threads. The distinction between lead and pitch should be carefully observed, as the two are often confounded, one with the other.

The thread may be single, double, triple, etc., the index of the thread i, being 1, 2, 3, etc., in accordance therewith. The relation between lead and pitch may then be expressed by an equation, thus:

$$L = i\,P^1. \qquad \textbf{(84)}$$

When the index of the thread is changed the speed ratio is changed, the relation being shown by the equation:

$$\frac{M}{M_w} = \frac{i}{N} \qquad \textbf{(85)}$$

If the efficiency were 100 per cent between the two shafts, the twisting moments would be inversely as the ratio of the speeds thus:

$$\frac{T_w}{T} = \frac{M}{M_w} = \frac{i}{N};$$

or,
$$T_w = \frac{Ti}{N}; \qquad (86)$$

but for an efficiency E the equation would be:

$$\frac{T_w}{T} = \frac{i}{EN};$$

or,
$$T_w = \frac{Ti}{EN}. \qquad (87)$$

The diameter of the worm is arbitrary. Change of this diameter has no effect on the speed ratio. It has a slight effect on the efficiency, the smaller worm giving a little higher efficiency. The diameter of the worm runs ordinarily from 3 to 10 times the circular pitch, an average value being $4P^1$ or $5P^1$.

A longitudinal cross-section through the axis of the worm cuts out a rack tooth, and this tooth section is usually made of the standard $14\frac{1}{2}°$ involute form shown in Fig. 46 for a rack.

The end thrust, of a magnitude practically equal to the pressure between the teeth, has to be taken by the hub of the worm against the face of the shaft bearing. A serious loss of efficiency from friction is likely to occur here. This is often reduced, however, by roller or ball bearings. With two worms on the same shaft, each driving into a separate worm gear, it is possible to make one of the worms right-hand thread, and the other left-hand, in which case the thrust is self-contained in the shaft itself, and there is absolutely no end thrust against the face of the bearing. This involves a double outfit throughout, and is not always practicable.

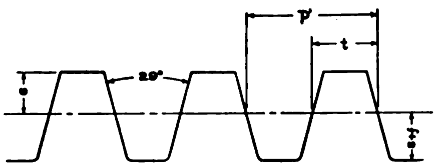


Fig. 46.

There are few mathematical equations necessary for the dimensioning of a worm and worm gear. The formulæ for the tooth parts as given on page 120 apply equally well in this case.

PRACTICAL MODIFICATION. The discussion of the efficiency E of the worm and worm gear is more of a practical than

of a theoretical nature. It seems to be true from actual operation, as well as theory, that the steeper the threads the higher the efficiency. In actual practice we seldom have opportunity to change the slope of the thread to get increased efficiency. The slope is usually settled from considerations of speed ratio, or available space, or some other condition. The usual practical problem is to take a given worm and worm gear, and to make out of it as efficient a device as possible. With hobbed gears running in oil baths, and with moderate pressures and speeds, the efficiency will range between 40 per cent and 70 per cent. The latter figure is higher than is usually attained.

To avoid cutting and to secure high efficiency, it seems essential to make the worm and the gear of different materials. The worm-thread surfaces being in contact a greater number of times than the gear teeth, should evidently be of the harder material. Hence we usually find the worm of steel, and the gear of cast iron, brass, or bronze. To save the expense of a large and heavy bronze gear, it is common to make a cast-iron center and bolt a bronze rim to it.

The worm, being the most liable to replacement from wear, it is desirable so to arrange its shaft fastening and general accessibility that it may be readily removed without disturbing the worm gear.

The circular pitch of the gear and the pitch of the worm thread must be the same, and the practical question comes in as to the threads per inch possible to be cut in the lathe in the production of the worm thread. The pitch must satisfy this requirement; hence the pitch will usually be fractional, and the diameter of the worm gear, to give the necessary number of teeth, must be brought to it. While it would perhaps be desirable to keep an even diametral pitch for the worm gear, yet it would be poor design to specify a worm thread which could not be cut in a lathe.

The standard involute of $14\frac{1}{2}°$, and the standard proportions of teeth as given on page 120 are usually used for worm threads. This system requires the gear to have at least 30 teeth, for if fewer teeth are used the thread of the worm will interfere with the flanks of the gear teeth. This is a mathematical relation, and there are methods of preventing it by change of tooth proportions

or of angle of worm thread ; but there are few instances in which less than 30 teeth are required, and it is not deemed worth while to go into a lengthy discussion of this point.

The angle of the worm embraced by the worm-gear teeth varies from 60° to 90°, and the general dimensions of rim are made about the same as for spur gears. The arms, or the web, have the same reasons for their size and shape. Probably web gears and cross-shaped arms are more common than oval or elliptical sections.

Worm gears sometimes have cast teeth, but they are for the roughest service only, and give but a point bearing at the middle of the tooth. An accurately hobbed worm gear will give a bearing clear across the face of the tooth, and, if properly set up and cared for, makes a good mechanical device although admittedly of somewhat low efficiency.

Fig. 47 shows a detail drawing of a standard worm and worm gear. It should serve as a suggestion in design, and an illustration of the shop dimensions required for its production.

PROBLEMS ON SPUR, BEVEL, AND WORM GEARS.

1. Calculate proportions of a standard Brown & Sharpe gear tooth of $1\frac{1}{2}$ diametral pitch, making a rough sketch and putting the dimensions on it.

2. Suppose the above tooth to be loaded at the top with 5,000 lbs. If the face be 6 inches, calculate the fiber stress at the pitch line, due to bending.

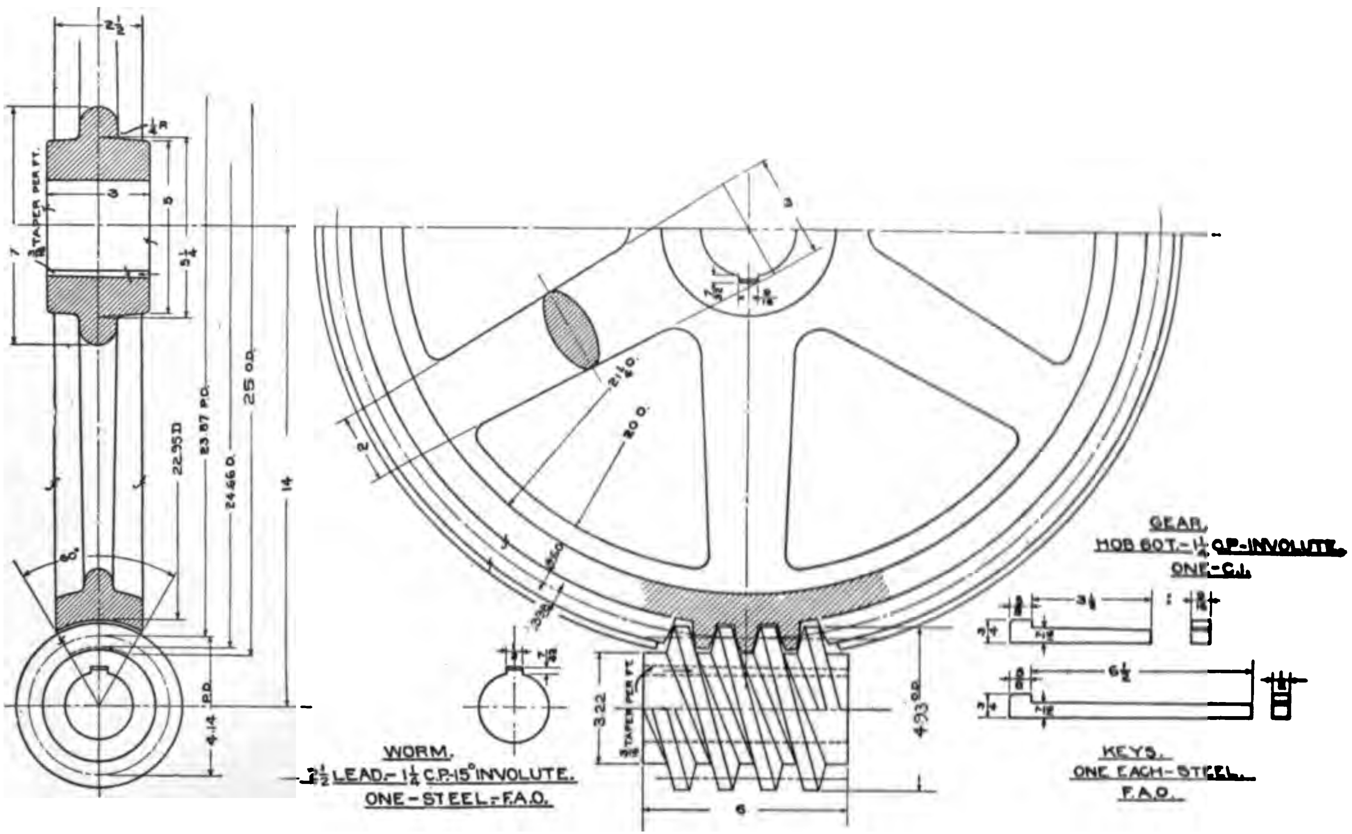
3. A tooth load of 1,200 lbs. is transmitted between two spur gears of 12-inch and 30-inch diameter, the latter gear making 100 revolutions per minute. Calculate a suitable pitch and face of tooth by the "Lewis" formula.

4. Assuming a $\frac{1}{2}$-inch web on the 12-inch gear, calculate the shearing fiber stress at the outside of a hub 4 inches in diameter

5. Design elliptical arms for the 30-inch gear, allowing $S = 2{,}200$.

6. Design cross-shaped arms for 30-inch gear.

7. Calculate the dimensions shown in formulæ 70 to 82 inclusive for a pair of bevel gears of 20 and 60 teeth respectively, 2 diametral pitch, and 4-inch face. (The use of logarithmic tables makes the calculation much easier than with the natural functions.)



SECTION AND ELEVATION OF WORM GEAR.
Fig. 47

8. A worm wheel has 40 teeth, 3 diametral pitch, and double thread. Calculate (*a*) its lead; (*b*) its pitch diameter.

FRICTION CLUTCHES.

NOTATION—The following notation is used throughout the chapter on Friction Clutches:.

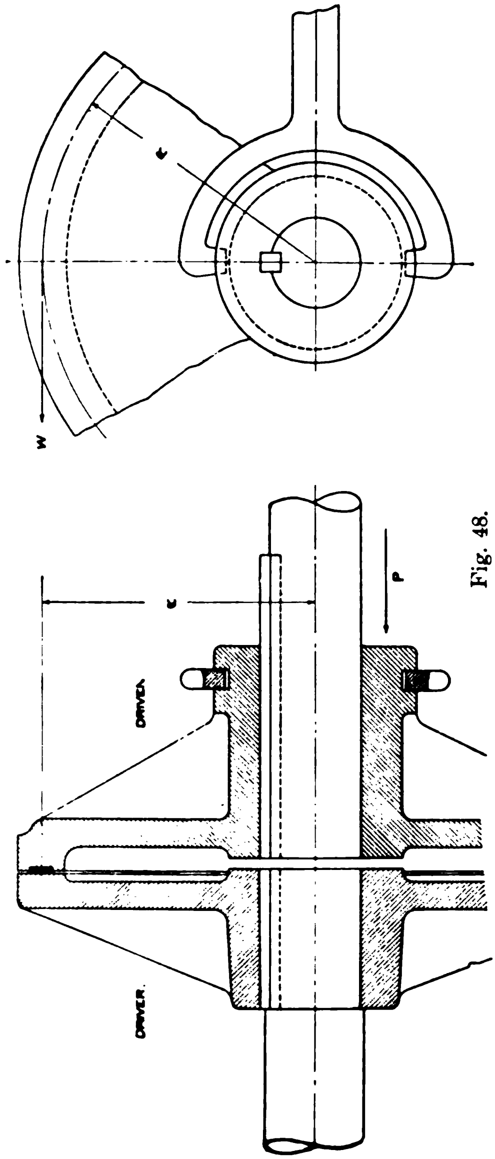


Fig. 48.

α = Angle between clutch face and axis of shaft (degrees)
H = Horse-power (33,000 ft.-lbs. per minute).
μ = Coefficient of friction (per cent).
N = Number of revolutions per minute.
P = Force to hold clutch in gear to produce W (lbs.).
R = Mean radius of friction surface (inches).
T = Twisting moment about shaft axis (inch-lbs.).
V = Force normal to clutch face (lbs.).
W = Load at mean radius of friction surface (lbs.).

ANALYSIS. The friction clutch is a device for connecting at will two separate pieces of shaft, transmitting an amount of power between them to the capacity of the clutch. The connection is usually accomplished while the driving shaft is under full speed, the slipping between the surfaces which occurs during the throwing-in of the clutch, permitting the driven shaft to pick up the speed of the other gradually, without appreciable shock. The disconnection is made in the same manner, the

amount of slipping which occurs depending on the suddenness with which the clutch is thrown out.

The force of friction is the sole driving element, hence the problem is to secure as large a force of friction as possible. But friction cannot be secured without a heavy normal pressure between surfaces having a high coefficient of friction between them. The many varieties of friction clutches which are on the market or designed for some special purpose, are all devices for accomplishing one and the same effect, *viz.*, the production of a heavy normal force or pressure between surfaces at such a radius from the driven axis, that the product of the force of friction thereby created and the radius shall equal the desired twisting moment about that axis.

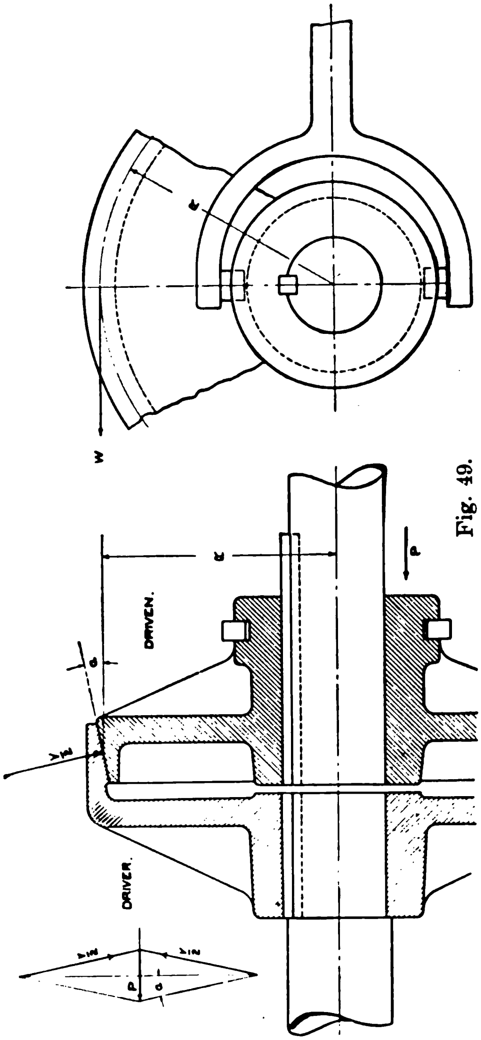


Fig. 49.

Three typical methods of accomplishing this are shown in Figs. 48, 49, and 50. None of these drawings is worked out in operative detail. They are merely illustrations of principle, and are drawn in the simplest form for that purpose.

In Fig. 48 the normal pressure is created in the simplest pos-

sible way, an absolutely direct push being exerted between the discs, due to the thrust P of the clutch fork.

In Fig. 49 advantage is taken of the wedge action of the inclined faces, the result being that it takes less thrust P to produce the required normal pressure at the radius R.

In Fig. 50 the inclination of the faces is carried so far that the angle a of Fig. 49 has become zero; and by the toggle-joint action of the link pivoted to the clutch collar, the normal force produced may be very great for a slight thrust P. By careful adjustment of the length of the link so that the jaw takes hold of the clutch surface, when the link stands nearly vertical, a very easy operating device is secured, and the thrust P is made a minimum.

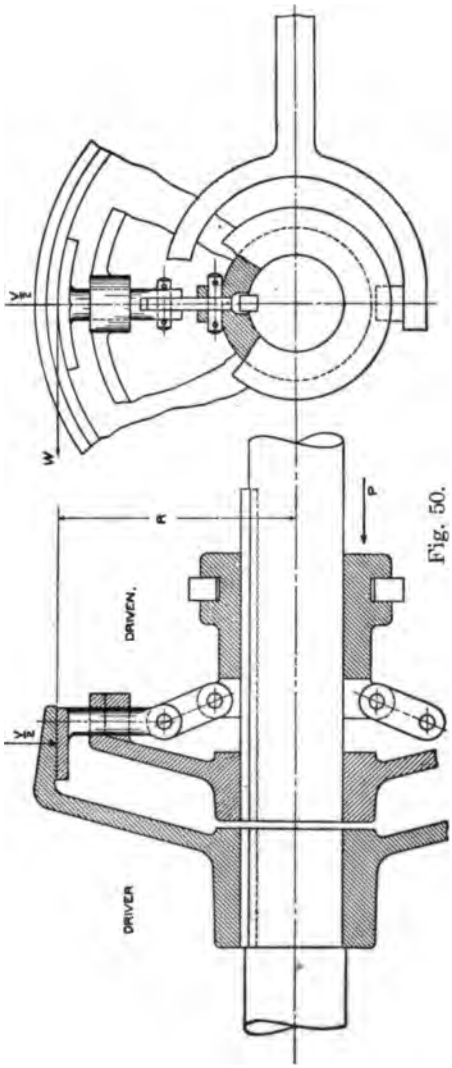


Fig. 50.

THEORY. Referring to Fig. 48 in order to calculate the twisting moment, we must remember that the force of friction between two surfaces is equal to the normal pressure times the coefficient of friction. This, in the form of an equation, using the symbols of the figure, is:

$$W = \mu P. \qquad \textbf{(88)}$$

Hence we may consider that we have a force of magnitude μP acting at the mean radius R of the clutch surface. The twisting moment will then be:

$$T = WR = \mu PR. \tag{89}$$

Referring to equation 54, which gives twisting moment in terms of horse-power, and putting the two expressions equal to each other, we have:

$$T = \frac{63{,}025H}{N} = \mu PR;$$

or,

$$H = \frac{\mu NPR}{63{,}025}. \tag{90}$$

This expression gives at once the horse-power that the clutch will transmit with a given end thrust P.

In Fig. 49 the equilibrium of the forces is shown in the little sketch at the left of the figure. The clutch faces are supposed to be in gear, and the extra force necessary to slide the two together is not considered, as it is of small importance. The static equations then are:

$$P = 2\frac{V}{2}\sin a;$$

or,

$$V = P \operatorname{cosec} a. \tag{91}$$

$$W = \mu V = \mu P \operatorname{cosec} a. \tag{92}$$

$$T = WR = \mu PR \operatorname{cosec} a. \tag{93}$$

$$T = \frac{63{,}025H}{N} = \mu PR \operatorname{cosec} a;$$

or,

$$H = \frac{\mu NPR \operatorname{cosec} a}{63{,}025}. \tag{94}$$

In Fig. 50, P would of course be variable, depending on the inclination of the little link. The amount of horse-power which this clutch would transmit would be the same as in the case of the device illustrated in Fig. 49, for an equal normal force V produced.

The further theoretical design of such clutches should be in accordance with the same principles as for arms and webs of pulleys, gears, etc. The length of the hubs must be liberal in

order to prevent tipping on the shaft as a result of uneven wear. The end thrust is apt to be considerable; and extra side stiffness must be provided, as well as a rim that will not spring under the radial pressure.

PRACTICAL MODIFICATION. It is desirable to make the most complicated part of a friction clutch the driven part, for then the mechanism requiring the closest attention and adjustment may be brought to and kept at rest when no transmission of power is desired.

Simplicity is an important practical requirement in clutches. The wearing surfaces are subjected to severe usage; and it is essential that they be made not only strong in the first place, but also capable of being readily replaced when worn out, as they are sure to be after some service.

The form of clutch shown in Fig. 50 is the most efficient form of the three shown, although its commercial design is considerably different from that indicated. Usually the jaws grip both sides of the rim, pinching it between them. This relieves the clutch rim of the radial unbalanced thrust.

Adjusting screws must be provided for taking up the wear, and lock nuts for maintaining their position.

Theoretically, the rubbing surfaces should be of those materials whose coefficient of friction is the highest; but the practical question of wear comes in, and hence we usually find both surfaces of metal, cast iron being most common. For metal on metal the coefficient of friction μ cannot be safely assumed at more than 15 per cent, because the surfaces are sure to get oily.

A leather facing on one of the surfaces gives good results as to coefficient of friction, μ having a value, even for oily leather, of 20 per cent. Much slipping, however, is apt to burn the leather; and this is most likely to occur at high speeds.

Wood on cast iron gives a little higher coefficient of friction for an oily surface than metal on metal. Wood blocks can be so set into the face of the jaws as to be readily replaced when worn, and in such case make an excellent facing.

The angle a of a cone friction clutch of the type shown in Fig. 49, may evidently be made so small that the two parts will wedge together tightly with a very slight pressure P; or it may

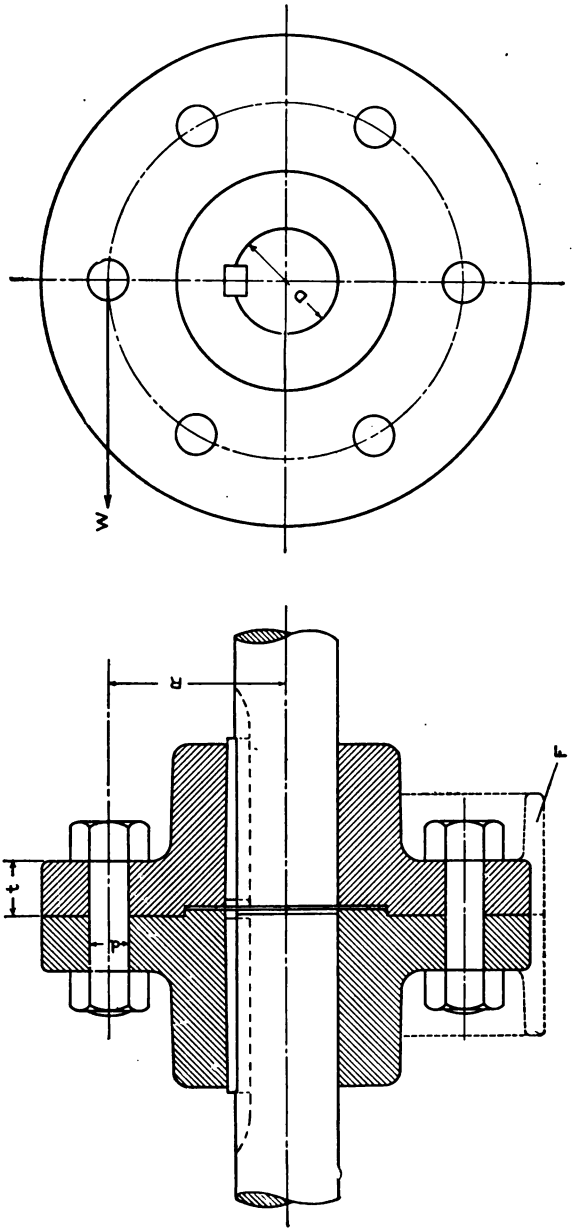


Fig. 51.

be so large as to have little wedging action, and approach the condition illustrated in Fig. 48. Between these limits there is a practical value which neither gives a wedging action so great as to make the surfaces difficult to pull apart, nor, on the other hand, requires an objectionable end thrust along the shaft in order to make the clutch drive properly.

For a = about 15°, the surfaces will free themselves when P is relieved.
" a = " 12°, " " require slight pull to be freed.
" a = " 10°, " " cannot be freed by direct pull of the hand, but require some leverage to produce the necessary force P.

PROBLEMS ON FRICTION CLUTCHES.

1. With what force must we hold a friction clutch in to transmit 30 horse-power at 200 revolutions per minute, assuming working radius of clutch to be 12 inches; coefficient of friction 15 per cent; angle $a = 10°$?

2. How much horse-power could be transmitted, other conditions remaining the same, if the working radius were increased to 18 inches ?

3. What force would be necessary in problem 1, if the angle a were 15°, other conditions remaining the same ?

COUPLINGS.

NOTATION.—The following notation is used throughout the chapter on Couplings:

D = Diameter of shaft (inches).
d = Diameter of bolt body (inches).
n = Number of bolts.
R = Radius of bolt circle (inches).
S = Safe shearing fiber stress (lbs. per sq. in.).
S_c = Safe crushing fiber stress (lbs. per sq. in.).
T = Twisting moment (inch-lbs.).
t = Thickness of flange (inches).
W = Load on bolts (lbs.).

ANALYSIS. Rigid couplings are intended to make the shafts which they connect act as a solid, continuous shaft. In order that the shaft may be worked up to its full strength capacity, the coupling must be as strong in all respects as the shaft, or, in other words, it must transmit the same torsional moment. In the analysis of the forces which come upon these couplings, it is not considered that they are to take any side load, but that they are to act purely as torsional elements. It is doubtless true that in many cases they do have to provide some side strength and stiffness, but this is not their natural function, nor the one upon which their design is based.

Referring to Fig. 51, which is the type most convenient for analysis, we have an example of the simplest form of **flange coupling.** It consists merely of hubs keyed to the two portions, with flanges driving through shear on a series of bolts arranged concentrically about the shaft. The hubs, keys, and flanges are subject to the same conditions of design as the hubs, keys, and web of a gear or pulley, the key tending to shear and be crushed in the hub and shaft, and the hub tending to be torn or sheared from the flange. The driving bolts, which must be carefully fitted in reamed holes, are subject to a purely shearing stress over their full area at the joint, and at the same time tend to crush the metal in the flange, against which they bear, over their projected area. This latter stress is seldom of importance, the thickness of the flange, for practical reasons, being sufficient to make the crushing stress very low.

THEORY. The theory of hubs, keys, and flanges, being like that already given for pulleys and gears, need not be repeated for couplings. The shearing stress on the bolts is the only new point to be studied.

In Fig. 51, for a twisting moment on the shaft of T, the load at the bolt circle is $W = \frac{T}{R}$. If the number of bolts be n, equating the external force to the internal strength, we have:

$$W = \frac{T}{R} = \frac{S\pi d^2}{4} n. \qquad (95)$$

Although the crushing will seldom be of importance, yet for the sake of completeness its equation is given, thus:

$$W = \frac{T}{R} = S_c dtn. \qquad (96)$$

The internal moment of resistance of the shaft is $\frac{SD^3}{5.1}$; hence the equation representing full equality of strength between the shaft and the coupling, depending upon the shearing strength of the bolts, is:

$$\frac{SD^3}{5.1R} = \frac{S\pi d^2}{4} n. \qquad (97)$$

The theory of the other types of couplings is obscure, except as regards the proportions of the key, which are the same in all cases. The shell of the clamp coupling, Fig. 52, should be thick enough to give equal torsional strength with the shaft; but the exact function which the bolts perform is difficult to determine. In general the bolts clamp the coupling tightly on the shaft and provide rigidity, but the key does the principal amount of the driving. The bolt sizes, in these couplings, are based on judgment and relation to surrounding parts, rather than on theory.

PRACTICAL MODIFICATION. All couplings must be made with care and nicely fitted, for their tendency, otherwise, is to

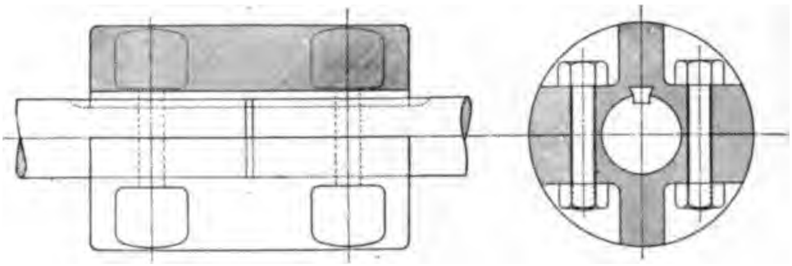


Fig. 52.

spring the shafts out of line. In the case of the flange coupling, the two halves may be keyed in place on the shafts, the latter then swung on centers in the lathe, and the joint faced off. Thus the joint will be true to the axis of the shaft; and, when it is clamped in position by the bolts, no springing out of line can take place.

A flange F (see Fig. 51) is sometimes made on this form of coupling, in order to guard the bolts. It may be used, also, to take a light belt for driving machinery; but a side load is thereby thrown on the shaft at the joint, which is at the very point where it is desirable to avoid it.

The simplest form of rigid coupling consists of a plain sleeve slipped over from one shaft to the other, when the second is butted up against the first. This is known as a **muff coupling.** When once in place, this is a very excellent coupling, as it is perfectly smooth on the outside, and consists of the fewest possible parts, merely a sleeve and a key. It is, however, expensive to fit,

difficult to remove, and requires an extra space of half its length on the shaft over which to be slipped back.

The **clamp coupling** is a good coupling for moderate-sized shafts, where the flange type of Fig. 51 would be unnecessarily expensive. The clamp coupling, Fig. 52, is simply a muff coupling split in halves, and recessed for bolts. It is cheap and is easily applied and removed, even with a crowded shaft. If bored with a piece of paper in the joint, when it is clamped in position it will pinch the shaft tightly and make a rigid connection. It is desirable to have the bolt-heads protected as much as possible, and this may be accomplished by making the outside diameter large enough so that the bolts will not project. Often an additional shell is provided to encase the coupling completely after it is located.

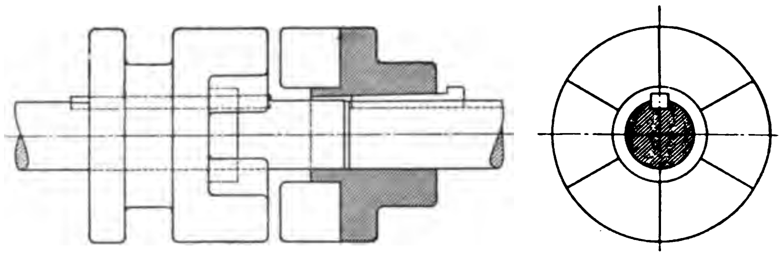


Fig. 53.

There are many other special forms of couplings, some of them adjustable. Most of them depend upon a wedging action exerted by taper cones, screws, or keys. Trade catalogues are to be sought for their description.

The **claw coupling**, Fig. 53, is nothing but a heavy flange coupling with interlocking claws or jaws on the faces of the flanges, to take the place of the driving bolts. This coupling can be thrown in or out as desired, although it usually performs the service of a rigid coupling, as it is not suited to clutching-in during rapid motion, like a friction clutch.

Flexible couplings, which allow slight lack of alignment, are made by introducing between the flanges of a coupling a flexible disc, the one flange being fastened to the inner circle of the disc, the other to the outer circle. This is also accomplished by providing the faces of the flange coupling with pins that drive by

pressure together or through leather straps wrapped round the pins. These devices are mostly of a special and often uncertain nature, lacking the positiveness which is one essential feature of a good coupling.

PROBLEMS ON COUPLINGS.

1. A flange coupling of the type of Fig. 51 is usea on a shaft 2 inches is diameter. The hub is 3 inches long, and carries a standard key, of proportions indicated below in the table of "Proportions for Gib Keys" (page 166). The bolt circle is 7 inches in diameter, and it is desired to use $\frac{5}{8}$-inch bolts. How many bolts are needed to transmit 60,000 inch-lbs., for a fiber stress in the bolt of 6,000?

2. Using 6 bolts, what diameter of bolt would be required?

3. If four $\frac{3}{4}$-inch bolts were used on a circle of 8 inches diameter, what diameter of shaft would be used in the coupling to give equal strength with the bolts?

BOLTS, STUDS, NUTS, AND SCREWS.

NOTATION—The following notation is used throughout the chapter on Bolts, Studs, Nuts, and Screws:

d = Diameter of bolt (inches).
d_1 = Diameter at root of thread (inches).
H = Height of nut (inches).
I = Initial axial tension (lbs.).
k = Allowable bearing pressure on surface of thread (lbs. per sq. in.).
L = Lead, or distance nut advances along axis in one revolution (inches).
l = Length of wrench handle (inches)
n = Number of threads in nut $= \frac{H}{p}$.
P = Axial load (lbs.).
p = Pitch of thread, or distance between similar points on adjacent threads, measured parallel to axis (inches).
S = Fiber stress (lbs. per sq. in.).
W = Load on bolt (lbs.).

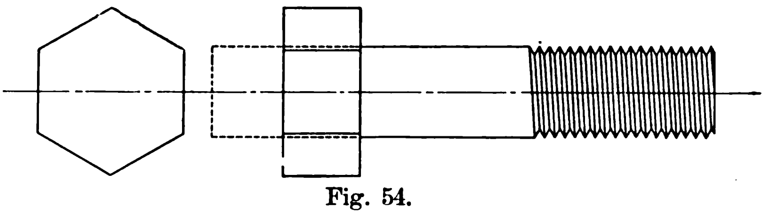


Fig. 54.

ANALYSIS. A **bolt** is simply a cylindrical bar of metal upset at one end to form a head, and having a thread at the other end, Fig. 54. A **stud** is a bolt in which the head is replaced by a thread; or it is a cylindrical bar threaded at both ends, usually

having a small plain portion in the middle, Fig. 55. The object of bolts and studs is to clamp machine parts together, and yet permit these same parts to be readily disconnected. The bolt passes through the pieces to be connected, and, when tightened, causes surface compression between the parts, while the reactions on the head and nut produce tension in the bolt. Studs and tap bolts pass through one of the connected parts and are screwed into the other, the stud remaining in position when the parts are disconnected.

As all materials are elastic within certain limits, the action of

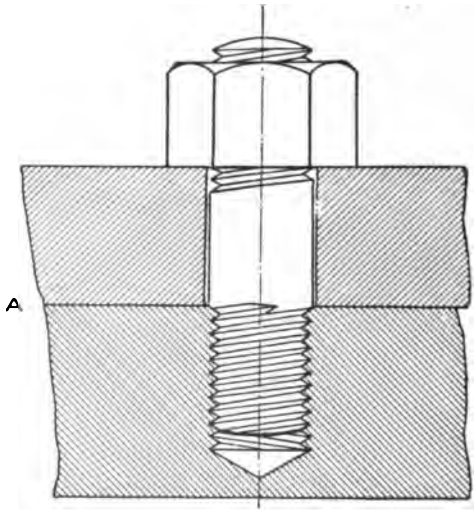


Fig. 55.

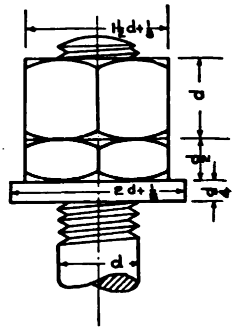


Fig. 55*a*.

a bolt in clamping two machine parts together, more especially if there is an elastic packing between them, may be represented diagrammatically by Fig. 56, in which a spring has been introduced to take the compression due to screwing up the nut. Evidently the tension in the bolt is equal to the force necessary to compress the spring. Now, suppose that two weights, each equal to $\frac{1}{2}$ W, are placed symmetrically on either side of the bolt, then the tension in the bolt will be increased by the added weights if the bolt is perfectly rigid. The bolt, however, stretches; hence some of the compression on the spring is relieved and the total tension in the

bolt is less than W + I, by an amount depending on the relative elasticity of the bolt and spring.

Suppose that the stud in Fig. 55 is one of the studs connecting the cover to the cylinder of a steam engine, and that the studs have a small initial tension; then the pressure of the steam loads each stud, and, if the studs stretch enough to relieve the initial pressure between the two surfaces, then their stress is due to the steam pressure only; or, from Fig. 56, when I = W; the initial pressure due to the elasticity of the joint is entirely relieved by the assumed stretch of the studs. Except to prevent leakage, it is seldom necessary to consider the initial tension, for the stretch of the bolt may be counted on to relieve this force, and the working tension on the bolt is simply the load applied.

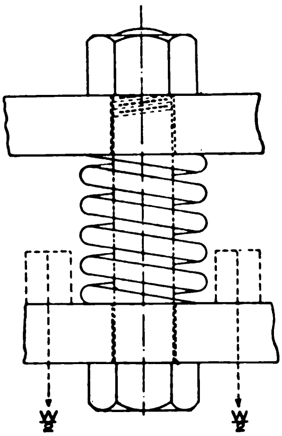


Fig. 56.

For shocks or blows, as in the case of the bolts found on the marine type of connecting-rod end, the stretch of the bolts acts like a spring to reduce the resulting tensions. So important is this feature that the body of the bolt is frequently turned down to the diameter of the bottom of the thread, thus uniformly distributing the stretch through the full length of the bolt, instead of localizing it at the threaded parts.

In tightening up a bolt, the friction at the surface of the thread produces a twisting moment, which increases the stress in the bolts, just as in the case of shafting under combined tension and torsion; but the increase is small in amount, and may readily be taken care of by permitting low values only for the fiber stress.

In a flange coupling, bolts are acted upon by forces perpendicular to the axis, and hence are under pure shearing stress. If the torque on the shaft becomes too great, failure will occur by the bolts shearing off at the joint of the coupling.

A bolt under tension communicates its load to the nut through the locking of the threads together. If the nut is thin, and the number of threads to take the load few, the threads may break or

shear off at the root. With a V thread there is produced a component force, perpendicular to the axis of the bolt, which tends to split the nut.

In screws for continuous transmission of motion and power, the thread may be compared to a rough inclined plane, on which a small block, the nut, is being pushed upward by a force parallel to the base of the plane. The angle at the bottom of the plane is the angle of the helix, or an angle whose tangent is the lead divided by the circumference of the screw. The horizontal force corresponds to the tangential force on the screw. The friction at the surface of the thread produces a twisting moment about the axis of the screw, which, combined with the axial load, subjects the screw to combined tension and torsion. Screws with square threads are generally used for this service, the sides of the thread exerting no bursting pressure on the nut. The proportions of screw thread for transmission of power depend more on the bearing pressure than on strength. If the bearing surface be too small and lubrication poor, the screw will cut and wear rapidly.

THEORY. A direct tensile stress is induced in a bolt when it carries a load exerted along its axis. This load must be taken by the section of the bolt at the bottom of the thread. If the area at the root of the thread is $\frac{\pi d_1^2}{4}$, and if S is the allowable stress per square inch, then the internal resistance of the bolt is $\frac{S\pi d_1^2}{4}$. Equating the external load to the internal strength we have:

$$W = \frac{S\pi d_1^2}{4}. \qquad (98)$$

For bolts which are used to clamp two machine parts together so that they will not separate under the action of an applied load, the initial tension of the bolt must be at least equal to the applied load. If the applied load is W, then the parts are just about to separate when $I = W$. Therefore the above relation for strength is applicable. As the initial tension to prevent separation should be a little greater than W, a value of S should be chosen so that there will be a margin of safety. For ordinary wrought iron and steel, S may be taken at 6,000 to 8,000.

If, however, the joints must be such that there is no leakage between the surfaces, as in the case of a steam cylinder head, and supposing that elastic packings are placed in the joints, then a much larger margin should be made, for the maximum load which may come on the bolt is I + W, where W is the proportional share of the internal pressure carried by the bolt. In such cases S = 3,000 to 5,000, using the lower value for bolts of less than $\frac{3}{4}$-inch diameter.

The table given on page 154 will be found very useful in proportioning bolts with U. S. standard thread for any desired fiber stress.

To find the initial tension due to screwing up the nut, we

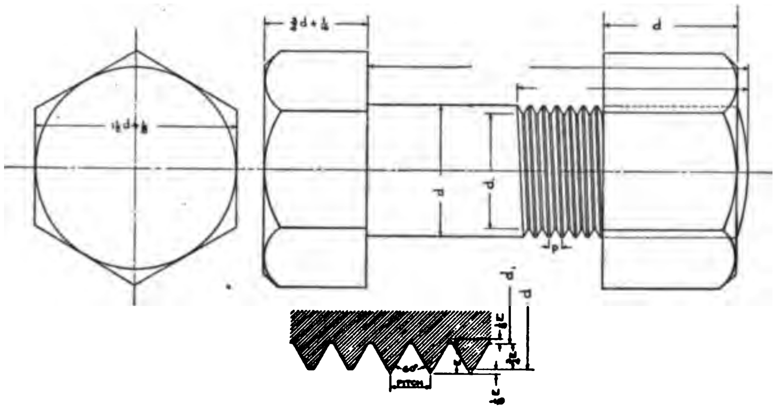


Fig. 56*a*.

may assume the length of the handle of an ordinary wrench, measured from the center of the bolt, as about 16 times the diameter of the bolt. For one turn of the wrench a force F at the handle would pass over a distance $2\pi l$, and the work done is equal to the product of the force and space, or $F \times 2\pi l$. At the same time the axial load P would be moved a distance p along the axis. Assuming that there is no friction, the equation for the equality of the work at the handle and at the screw is:

$$F2\pi l = Pp. \qquad (99)$$

Friction, however, is always present; hence the ratio of the useful work (Pp) to the work applied ($F2\pi l$) is not unity as above re-

TABLE FOR STRENGTH OF BOLTS.—U. S. Standard Thread.

Bolt.		Diameters.		Areas.		Approximate Tensile Strength. at Bottom of Thread. (In Hundreds of Pounds.)					Approximate Shearing Strength. Full Bolt Diameter. (In Hundreds of Pounds.)			
Diameter. Inches.	Threads per Inch.	Bottom of Thread.	Tap Drill.	Bolt Body.	Bottom of Thread.	At 4,000 lbs. per sq. in.	At 5,000 lbs. per sq. in.	At 6,000 lbs. per sq. in.	At 7,000 lbs. per sq. in.	At 10,000 lbs. per. sq. in.	At 4,000 lbs. per sq. in.	At 5,000 lbs. per sq. in.	At 6,000 lbs. per sq. in.	At 7,000 lbs. per sq. in.
¼	20	.18	3/16	.05	.03	1.08	1.35	1.60	1.88	2.69	1.96	2.45	2.95	3.43
5/16	18	.24	¼	.08	.04	1.82	2.27	2.72	3.18	4.54	3.01	3.83	4.60	5.37
⅜	16	.29	5/16	.11	.07	2.71	3.39	4.07	4.75	6.78	4.40	5.52	6.62	7.73
7/16	14	.34	23/64	.15	.09	3.73	4.67	5.60	6.53	9.33	6.00	7.51	9.02	11.00
½	13	.40	13/32	.20	.13	5.00	6.25	7.50	8.75	12.00	7.84	9.81	12.00	14
9/16	12	.45	15/32	.25	.16	6.48	8.10	9.62	11.00	16	9.92	12.00	15	17
⅝	11	.51	17/32	.31	.20	8.04	10.00	12.00	14	20	12.00	15	18	21
¾	10	.62	⅝	.44	.30	12.00	15	18	21	30	18	22	26	31
⅞	9	.73	¾	.60	.42	17	24	25	29	42	24	30	36	42
1	8	.84	27/32	.78	.55	22	27	33	38	55	31	39	47	55
1⅛	7	.94	31/32	.99	.69	28	34	41	48	69	40	50	60	70
1¼	7	1.06	1 3/32	1.23	.89	31	39	47	55	78	49	61	74	86
1⅜	6	1.16	1 3/16	1.48	1.05	42	53	64	72	106	59	74	89	104
1½	6	1.28	1 9/32	1.77	1.29	51	64	77	90	128	71	88	106	124
1⅝	5½	1.39	1 13/32	2.07	1.51	61	78	92	100	153	83	104	124	145
1¾	5	1.49	1½	2.40	1.74	70	88	106	123	176	96	120	144	168
1⅞	5	1.61	1⅝	2.76	2.05	81	101	122	142	203	110	138	166	193
2	4½	1.71	1¾	3.14	2.30	92	115	138	161	230	126	157	186	220
2¼	4½	1.96	1 31/32	3.98	3.02	125	156	187	218	312	159	199	238	278
2½	4	2.17	2 3/16	4.91	3.72	148	185	222	259	370	196	245	294	344
2¾	4	2.42	2 7/16	5.94	4.62	184	230	276	322	460	237	297	356	416
3	3½	2.63	2⅝	7.07	5.43	218	272	326	381	544	283	353	424	495
3¼	3½	2.88	2 29/32	8.29	6.51	264	330	396	462	660	332	415	498	581
3½	3¼	3.10	3 1/32	9.62	7.55	302	376	452	528	754	385	481	577	673
3¾	3	3.32	3 11/32	11.04	8.64	344	430	516	602	860	442	552	663	773
4	3	3.57	3 19/32	12.57	9.99	396	495	594	693	990	503	628	754	880
4¼	2⅞	3.80	3 13/16	14.19	11.33	452	565	678	791	1,130	567	709	851	993
4½	2¾	4.03	4 1/32	15.90	12.74	507	634	760	888	1,268	636	795	954	1,113
4¾	2⅝	4.25	4 9/32	17.72	14.22	567	709	851	1,993	1,420	709	886	1,063	1,240
5	2½	4.48	4½	19.63	15.76	630	788	946	1,103	1,676	785	982	1,178	1,374
5½	2⅜	4.95	4 31/32	23.76	19.27	770	962	1,154	1,347	1,924	950	1,188	1,425	1,663
6	2¼	5.42	5 7/16	28.27	23.09	923	1,159	1,384	1,617	2,307	1,128	1,414	1,696	1,979

lations assume. From numerous experiments on the friction of screws and nuts, it has been found that the efficiency may be as low as 10 per cent. Introducing the efficiency in above equation, it may be written:

$$\frac{Pp}{F2\pi l} = \frac{1}{10}. \qquad (100)$$

Assuming that 50 pounds is exerted by a workman in

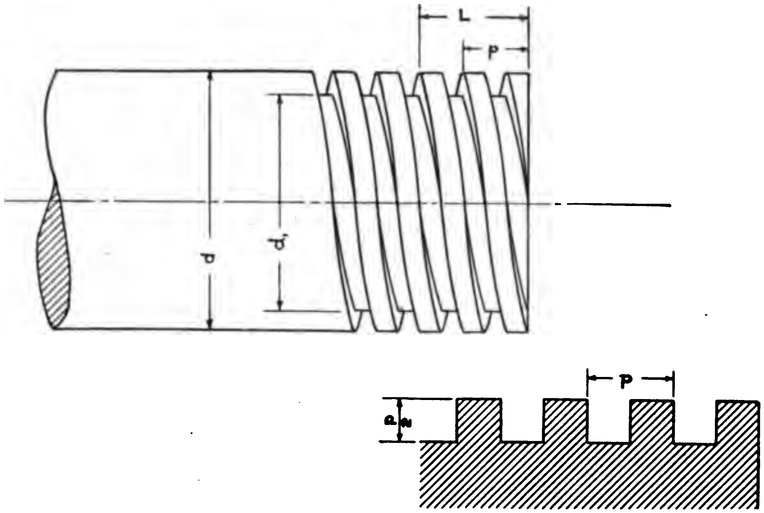


Fig. 58.

tightening up the nut on a 1-inch bolt, the equation above shows that $P = 4{,}021$ pounds; or the initial tension is somewhat less than the tabular safe load shown for a 1-inch bolt, with S assumed at 10,000 pounds per sq. inch.

For shearing stresses the bolt should be fitted so that the body of the bolt, not the threads, resists the force tending to shear off the bolt perpendicular to its axis. The internal strength of the bolt to resist shear is the allowable stress S times the area of the bolt in shear, or $\frac{S\pi d^2}{4}$. If W represents the external force tending to shear the bolt the equality of the external force to the internal strength is:

$$W = \frac{S\pi d^2}{4}. \qquad (101)$$

Reference to the table on page 154 for the shearing strength of bolts, may be made to save the labor of calculations.

Let Fig. 58 represent a square thread screw for the transmission of motion. The surface on which the axial pressure bears, if n is the number of threads in a nut, is $\frac{\pi}{4}(d^2 - d_1^2)\ n$. Suppose that a pressure of k pounds per square inch is allowed on the surface of the thread. Then the greatest permissible axial load P must not exceed the allowable pressure; or, equating,

$$P = k \frac{\pi}{4}(d^2 - d_1^2)\ n \qquad (102)$$

The value of k varies with the service required. If the motion be slow and the lubrication very good, k may be as high as 900. For rapid motion and doubtful lubrication, k may not be over 200. Between these extremes the designer must use his judgment, remembering that the higher the speed the lower is the allowable bearing pressure.

PRACTICAL MODIFICATION. It will be noted in the formulæ for bolt strengths that different values for S are assumed. This is necessary on account of the uncertain initial stresses which are produced is setting up the nuts. For cases of mere fastening, the safe tension is high, as just before the joint opens the tension is about equal to the load and yet the fastening is secure. On the other hand, bolts or studs fastening joints subjected to internal fluid pressure must be stressed initially to a greater amount than the working pressure which is to come on the bolt. As this initial stress is a matter of judgment on the part of the workman, the designer, in order to be on the safe side, should specify not less than $\frac{5}{8}$-inch or $\frac{3}{4}$-inch bolts for ordinary work, so that the bolts may not be broken off by a careless workman accidentally putting a greater force than necessary on the wrench handle. In making a steam-tight joint, the spacing of the bolts will generally determine their number; hence we often find an excess of bolt strength in joints of this character.

Through bolts are preferred to studs, and studs to tap bolts or cap screws. If possible, the design should be such that through bolts may be used. They are cheapest, are always in standard

stock, and well resist rough usage in connecting and disconnecting. The threads in cast iron are weak and have a tendency to crumble; and if a through bolt cannot be used in such a case, a stud, which can be placed in position once for all, should be employed—not a tap bolt, which injures the thread in the casting every time it is removed.

The plain portion of a stud should be screwed up tight against the shoulder, and the tapped hole should be deep enough to prevent bottoming. To avoid breaking off the stud at the shoulder, a neck, or groove, may be made at the lower end of the thread entering the nut.

To withstand shearing forces the bolts must be fitted so that no lost motion may occur, otherwise pure shearing will not be secured.

Nuts are generally made hexagonal, but for rough work are often made square. The hexagonal nut allows the wrench to turn through a smaller angle in tightening up, and is preferred to the square nut. Experiments and calculations show that the height of the nut with standard threads may be about $\frac{1}{2}$ the diameter of the bolt and still have the shearing strength of the thread equal to the tensile strength of the bolt at the root of the thread. Practically, however, it is difficult to apply such a thin wrench as this proportion would call for on ordinary bolts. More commonly the height of the nut is made equal to the diameter of the bolt so that the length of thread will guide the nut on the bolt, give a low bearing pressure on the threads, and enable a suitable wrench to be easily applied. The standard proportions for bolts and nuts may be found in any handbook. Not all manufacturers conform to the United States standard; nor do manufacturers in all cases conform to one another in practice.

If the bolt is subject to vibration, the nuts have a tendency to loosen. A common method of preventing this is to use double nuts, or **lock nuts**, as they are called (see Fig. 55 A). The under nut is screwed tightly against the surface, and held by a wrench while the second nut is screwed down tightly against the first. The effect is to cause the threads of the upper nut to bear against the under sides of the threads of the bolt. The load on the bolt is sustained therefore by the upper nut, which should be the thicker

of the two; but for convenience in applying wrenches the position of the nuts is often reversed.

The form of thread adapted to transmitting power is the square thread, which, although giving less bursting pressure on the nut, is not as strong as the V thread for a given length, since the total section of thread at the bottom is only $\frac{1}{2}$ as great. If the pressure is to be transmitted in but one direction, the two

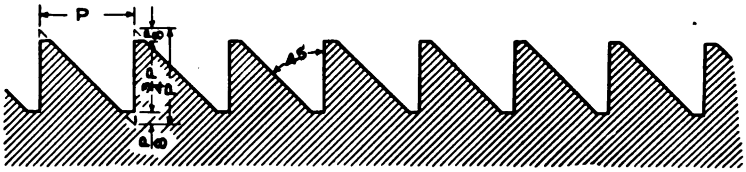


Fig. 59.

types may be combined advantageously to form the buttress thread of the proportions shown in Fig. 59. Often, as in the carriage of a lathe, to allow the split nut to be opened and closed over the lead screw, the sides of the thread are placed at a small angle, say 15°, to each other, as illustrated in Fig. 60.

The practical commercial forms in which we find screwed fastenings are included in five classes, as follows:

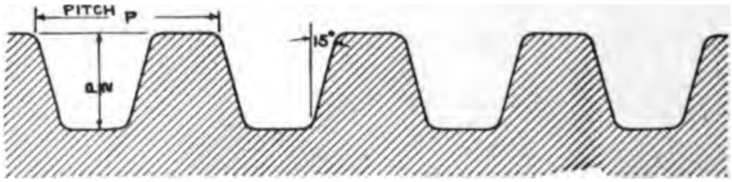


Fig. 60.

1. **Through bolts** (Fig. 61), usually rough stock, with square upset heads, and square or hexagonal nuts.

2. **Tap bolts** (Fig. 62), also called **cap screws.** These usually have hexagonal heads, and are found both in the rough form, and finished from the rolled hexagonal bar in the screw machine.

3. **Studs** (Fig. 63), rough or finished stock, threaded in the screw machine.

4. **Set screws** (Fig. 64), usually with square heads, and case-hardened points. Many varieties of set screws are made, the

principal distinguishing feature of each being in the shape of the point. Thus, in addition to the plain beveled point, we find the "cupped," rounded, conical, and "teat" points.

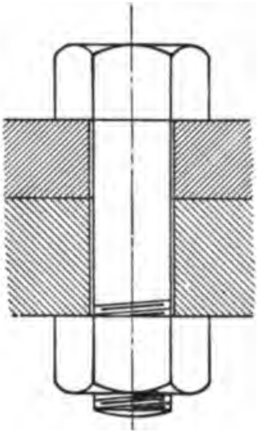


Fig. 61.

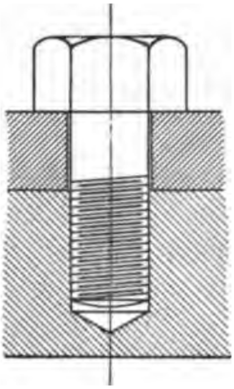


Fig. 62.

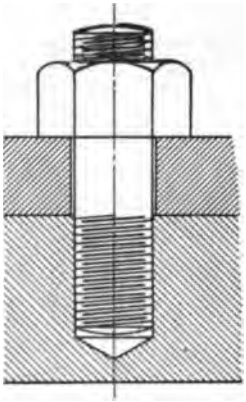


Fig. 63.

5. **Machine screws** (Fig. 64*a*), usually round, "button," or countersunk head. Common proportions are indicated relative to diameter of body of screw.

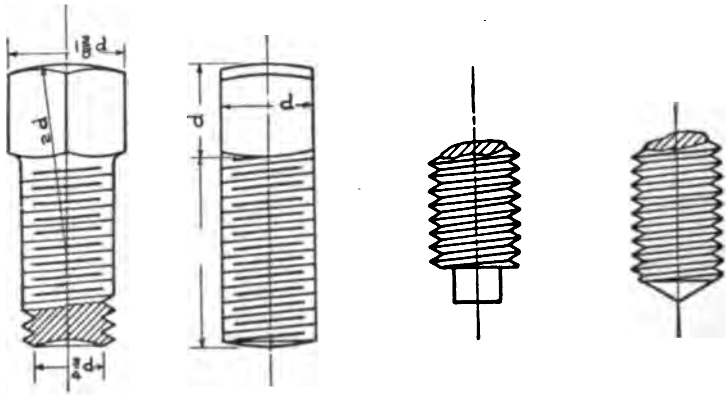


Fig. 64.

PROBLEMS ON BOLTS, STUDS, NUTS, AND SCREWS.

1. Calculate the diameter of a bolt to sustain a load of 5,000 lbs.

2. The shearing force to be resisted by each of the bolts of a flange coupling is 1,200 lbs. What commercial size of bolt is required?

3. With a wrench 16 times the diameter of the bolt, and an efficiency of 10 per cent, what axial load can a man exert on a standard $\frac{3}{4}$-inch bolt, if he pulls 40 lbs. at the end of the wrench handle?

4. A single, square-threaded screw of diameter 2 inches, lead $\frac{1}{4}$ inch, depth of thread $\frac{1}{8}$ inch, length of nut 3 inches, is to be allowed a bearing pressure of 300 lbs. per square inch. What axial load can be carried?

5. Calculate the shearing stress at the root of the thread in problem 4.

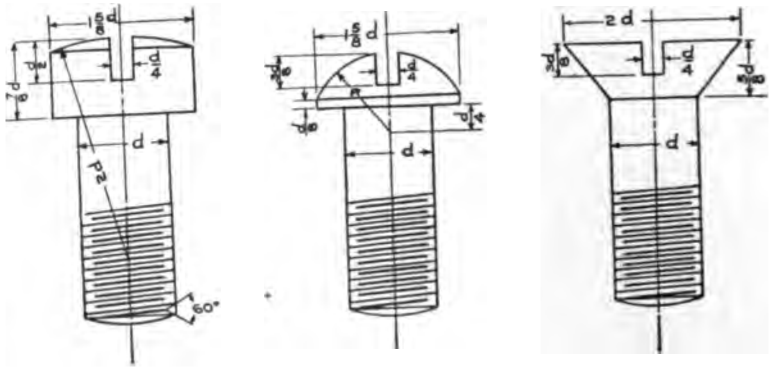


Fig. 64*a*.

KEYS, PINS, AND COTTERS.

NOTATION—The following notation is used throughout the chapter on Keys, Pins, and Cotters:

- D = Average diameter of rod (inches).
- D_1 = Outside diameter of socket (inches).
- d = Diameter of shaft (inches).
- L = Length of key (inches).
- P = Driving force (lbs.).
- P_1 = Axial load on rod (lbs.).
- R = Radius at which P acts (inches).
- S_c = Safe crushing fiber stress (lbs. per sq. in.).
- S_s = Safe shearing fiber stress (lbs. per sq. in.).
- S_t = Safe tensile fiber stress (lbs. per sq. in.).
- T = Thickness of key (inches).
- W = Width of key (inches).
- w = Average width of cotter (inches).
- w_1 = End of slot to end of rod (inches).
- w_2 = End of slot to end of socket (inches).

KEYS AND PINS.

ANALYSIS. Keys and pins are used to prevent relative

rotary motion between machine parts intended to act together as one piece. If we drill completely through a hub and across the shaft, and insert a tightly fitted pin, any rotary motion of the one will be transmitted to the other, provided the pin does not fail by shearing off at the joint between the shaft and the hub. The shearing area is the sum of the cross-sections of the pin at the joint.

We may drill a hole in the joint, the axis of the hole being parallel to the axis of the shaft, and drive in a pin, in which case we introduce a shearing area as before, but the area is now equal to the diameter of the pin multiplied by its length, and the pin is stressed sidewise, instead of across. It is evident in the sidewise case that we may increase the shearing area to anything we please, without changing the diameter of the pin, merely by increasing the length of the pin.

As there are some manufacturing reasons why a round pin placed lengthwise in the joint is not always applicable, we may make the pin a rectangular one, in which case it is called a **key.**

When pins are driven across the shaft as in the first instance, they are usually made taper. This is because it is easier to ream a taper hole to size than a straight hole, and a taper pin will drive more easily than a straight pin, it not being necessary to match the hole in hub and shaft so exactly in order that the pin may enter. The taper pin will draw the holes into line as it is driven, and can be backed out readily in removal.

Keys of the rectangular form are either straight or tapered, but for different reasons from those just stated for pins. Straight keys have working bearing only at the sides, driving purely by shear, crushing being exerted by the side of the key in both shaft and hub, over the area against the key. The key itself does not prevent end motion along the shaft; and if end motion is not desired, auxiliary means of some sort must be resorted to, as, for example, set screws through the hub jamming hard against the top of the key.

If end motion along the shaft is desired, the key is called a **spline**, and, while not jammed against the shaft, is yet prevented from changing its relation to the hub by some means such as illustrated in Fig. 65.

Taper keys not only drive through sidewise shearing strength, but prevent endwise motion by the wedging action exerted between the shaft and hub. These keys drive more like a strut from corner to corner; but this action is incidental rather than intentional, and the proportions of a taper key should be such that it will give its full resisting area in shearing and crushing, the same as a straight key.

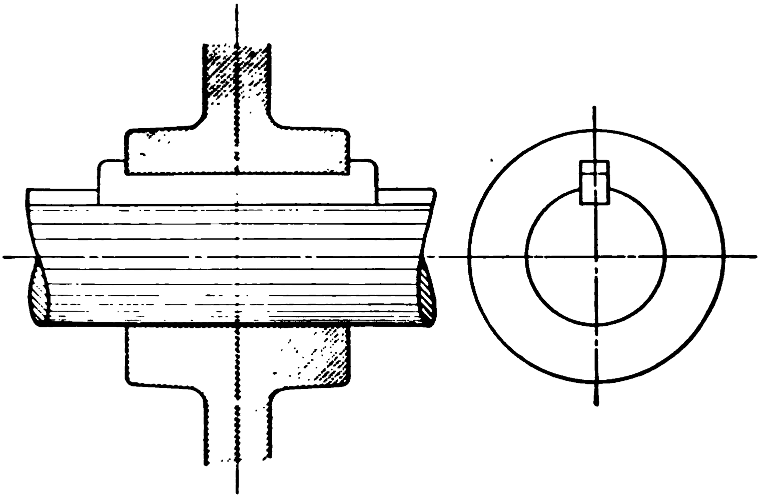


Fig. 65.

THEORY. Suppose that the pin illustrated in Fig. 66 passes through hub and shaft, and the driving force P acts at the radius R; then the force which is exerted at the surface of the shaft to shear off the pin at the points A and B is $\frac{2\,PR}{d}$. If D_1 is the average diameter of the pin, its shearing strength is $\frac{2\pi D_1^2 S_s}{4}$. Equating the external force to the internal strength, we have:

$$\frac{2PR}{d} = \frac{2\pi D_1^2 S_s}{4};$$

or,

$$D_1 = \sqrt{\frac{4PR}{\pi d S_s}}. \qquad (103)$$

In Fig. 67 a rectangular key is sunk half way in hub and shaft according to usual practice. Here the force at the surface

of the shaft, calculated the same as before, not only tends to shear off the key along the line AB, but tends to crush both the portion in the shaft and in the hub. The shearing strength along the

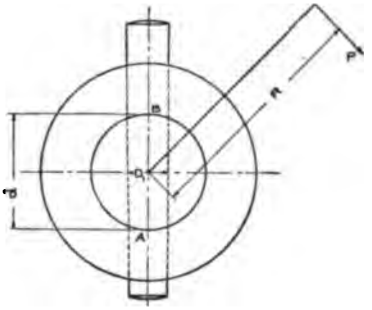


Fig. 66.

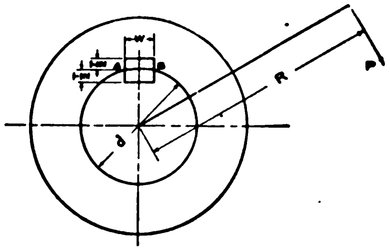


Fig. 67.

line AB is LWS_s. Equating external force to internal strength, we have:

$$\frac{2PR}{d} = LWS_s;$$

or,

$$W = \frac{2PR}{dLS_s}. \qquad (104)$$

The crushing strength is, of course, that due to the weaker metal, whether in shaft or hub. Let S_c be this least safe crushing fiber stress. The crushing strength then is $\frac{LT}{2}S_c$, and, equating external force to internal strength, we have:

$$\frac{2PR}{d} = \frac{LT}{2}S_c;$$

or,

$$T = \frac{4PR}{dLS_c}. \qquad (105)$$

The proportions of the key must be such that the equations as above, both for shearing and for crushing, shall be satisfied.

PRACTICAL MODIFICATION. Pins across the shaft can be used to drive light work only, for the shearing area cannot be very large. A large pin cuts away too much area of the shaft, decreasing the latter's strength, Pins are useful in preventing end motion, but in this case are expected to take no shear, and may be of small

diameter. The common split pin is especially adapted to this service, and is a standard commercial article.

Taper pins are usually listed according to the Morse standard taper, proportions of which may be found in any handbook. It is desirable to use *standard* taper pins in machine construction, as the reamers are a commercial article of accepted value, and readily obtainable in the machine-tool market.

With properly fitted keys, the shearing strength is usually the controlling element. For shafts of ordinary size, the standard proportions as given in tables like that below are safe enough without calculation, up to the limit of torsional strength of the shaft. For special cases of short hubs or heavy loads, a calculation is needed to check the size, and perhaps modify it.

Splines, also known as "feather keys," require thickness greater than regular keys, on account of the sliding at the sides. A table suggesting proportions for splines is given on page 166.

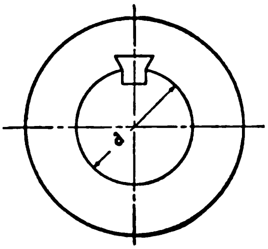


Fig. 67*a*.

Though the spline may be either in the shaft or hub, it is the more usual thing to find the spline dovetailed (Fig. 67*a*), "gibbed," or otherwise fastened in the hub; and a long spline way made in the shaft, in which it slides.

The straight key, accurately fitted, is the most desirable fastening device for accurate machines, such as machine tools, on account of the fact that there is absolutely no radial force exerted to throw the parts out of true. It, however, requires a tight fit of hub to shaft, as the key cannot be relied upon to take up any looseness.

The taper key (Fig. 68), by its wedging action, will take up some looseness, but in so doing throws the parts out slightly. Or, even if the bored fit be good, if the taper key be not driven home with care, it will spring the hub, and make the parts run untrue. The great advantage, however, that the taper key has of holding the hub from endwise motion, renders it a very useful and practical article. It is usually provided with a head, or "gib," which permits a draw hook to be used to wedge between the face of the hub and the key to facilitate starting the key from its seat.

Two keys at 90° from each other may be used in cases where one key will not suffice. The fine workmanship involved in spacing these keys so that they will drive equally makes this plan inadvisable except in case of positive and unavoidable necessity.

The "Woodruff" key (Fig. 69) is a useful patented article for certain locations. This key is a half-disc, sunk in the shaft

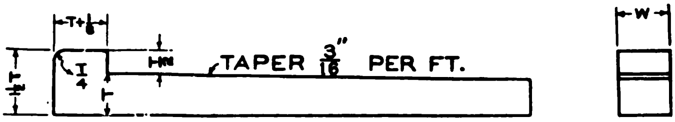


Fig. 68.

and the hub is slipped over it. A simple rotary cutter is dropped into the shaft to produce the key seat; and on account of the depth in the shaft, the tendency to rock sidewise is eliminated, and the drive is purely by shear.

Keys may be milled out of solid stock, or drop-forged to within a small fraction of finished size. The drop-forged key is an excellent modern production and requires but a minimum

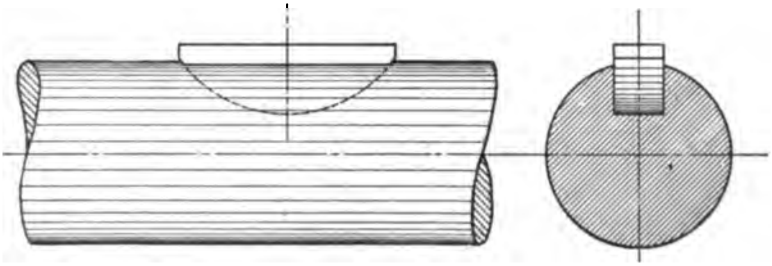


Fig. 69.

amount of fitting. Any key, no matter how produced, requires some hand fitting and draw filing to bring it properly to its seat and give it full bearing.

It is good mechanical policy to avoid keyed fastenings whenever possible. This does not mean that keys may never be used, but that a key is not an ideal way to produce an absolutely positive drive, partly because it is an expensive device, and partly because the tendency of any key is to work itself loose, even if carefully fitted.

The following tables are suggested as a guide to proportions

of gib keys and feather keys, and will be found useful in the absence of any manufacturer's standard list:

Fig. 70. **PROPORTIONS FOR GIB KEYS.**

Diameter of shaft (d), inches.	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{5}{8}$	2	$2\frac{1}{2}$	$3\frac{1}{4}$	4	5	$6\frac{1}{2}$
Width (W), inches.	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{5}{16}$	$1\frac{5}{8}$
Thickness (T), inches.	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{17}{32}$	$\frac{21}{32}$	$\frac{13}{16}$	1	$1\frac{1}{4}$

Fig. 71. **PROPORTIONS FOR FEATHER KEYS.**

Diameter of Shaft (d), inches.	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Width (W), inches.	$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{5}{8}$
Thickness (T), inches.	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$

COTTERS.

ANALYSIS. Cotters are used to fasten hubs to rods rather than shafts, the distinction between a rod and a shaft being that a rod takes its load in the direction of its length, and does not drive by rotation. A cotter, therefore, is nothing but a cross-pin of modified form, to take shearing and crushing stress in the direction of the axis of the rod, instead of perpendicular to it.

Referring to Fig. 72, one will see that the cotter is made long and thin—long, in order to get sufficient shearing area to resist shearing along lines A and B; thin, in order to cut as little cross-sectional area out of the body of the shaft as possible. The cotter itself tends to shear along the lines A and B, and crush along the surfaces K, G, and J. The socket tends to crush along the surfaces K and G. The rod end D tends to be sheared out along the lines C H and Q E, and also to be crushed along the surface J. The socket tends to be sheared along the lines V U and X Y.

The cotter is made taper on one side, thus enabling it to draw up the flange of the rod tightly against the head of the socket. This taper must not be great enough to permit easy "backing out" and loosening of the cotter under load or vibration in the rod. In responsible situations this cannot be safely guarded against except through some auxiliary locking device, such as lock nuts on the end of the cotter (Fig. 73).

THEORY. Referring to Fig. 72, assume an axial load of P_1, as shown. The successive equations of external force to internal

strength are enumerated below, for the different actions that take place:

For shearing along lines A and B, w being the average width of cotter, and S_s safe shearing stress of cotter,

$$P_1 = 2TwS_s. \qquad (106)$$

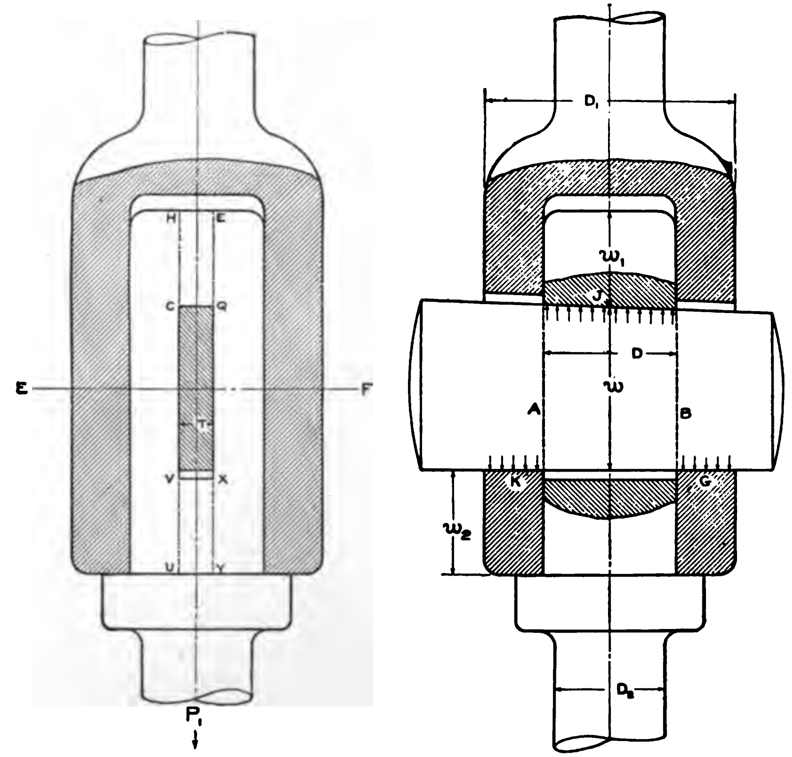


Fig. 72.

For crushing along surfaces K and G, S_c being least safe crushing stress, whether of cotter or socket,

$$P_1 = T(D_1 - D)S_c. \qquad (107)$$

For crushing along surface J, S_c being least safe crushing stress, whether of cotter or socket,

$$P_1 = DTS_c. \qquad (108)$$

For shearing along surfaces CH and QE, S_s being safe shearing stress of rod end, and w_1 end of slot to end of rod,

$$P_1 = 2w_1DS_s. \qquad \textbf{(109)}$$

For tension in rod end at section across slot, S_t being safe tensile stress in rod end,

$$P_1 = (\frac{\pi D^2}{4} - TD)S_t. \qquad \textbf{(110)}$$

For tension in socket at section across slot, S_t being safe tensile stress in socket,

$$P_1 = [\frac{\pi D_1^2}{4} - \frac{\pi D^2}{4} - T(D_1 - D)]S_t. \qquad \textbf{(111)}$$

For shearing in socket along the lines VU and XY, S_s being safe shearing stress in the socket, and w_2 end of slot to end of socket,

$$P_1 = 2w_2(D_1 - D)S_s. \qquad \textbf{(112)}$$

The proportions of cotter and socket may be fixed to some extent by practical or assumed conditions. The dimensions may then be tested by the above equations, that the safe working stresses may not be exceeded, the dimensions being then modified accordingly.

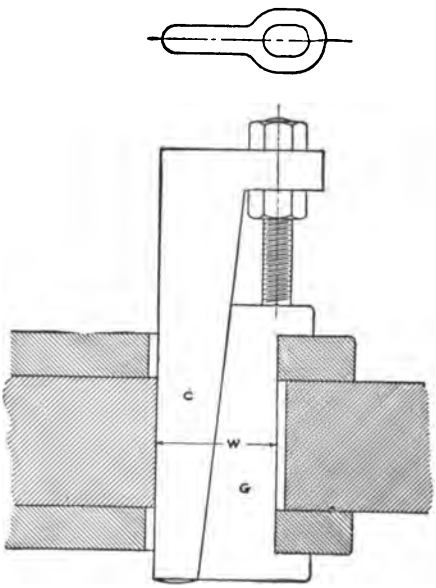


Fig. 73.

The steel of which both cotter and rod would ordinarily be made has range of working fiber stress as follows:

Tension, 8,000 to 12,000 (lbs. per sq. in.)
Compression, 10,000 to 16,000 (lbs. per sq. in.)
Shear, 6,000 to 10,000 (lbs. per sq. in.)

The socket, if made of cast iron, will be weak as regards tension, tendency to shear out at

the end, and tendency to split. The uncertainty of cast iron to resist these is so great that the hub or socket must be very clumsy in order to have enough surplus strength. This is always a noticeable feature of the cotter type of fastening, and cannot well be avoided.

PRACTICAL MODIFICATION. The driving faces of the cotter are often made semicircular. This not only gives more shearing area at the sides of the slots, but makes the production of the slots easier in the shop. It also avoids the general objection to sharp corners—namely, a tendency to start cracks.

A practicable taper for cotters is $\frac{1}{2}$ inch per foot. This will under ordinary circumstances prevent the cotter from backing out under the action of the load. When set screws against the side of the cotter, or lock nuts are used, as in Fig. 73, the taper may be greater than this, perhaps as much as $1\frac{1}{2}$ inches per foot.

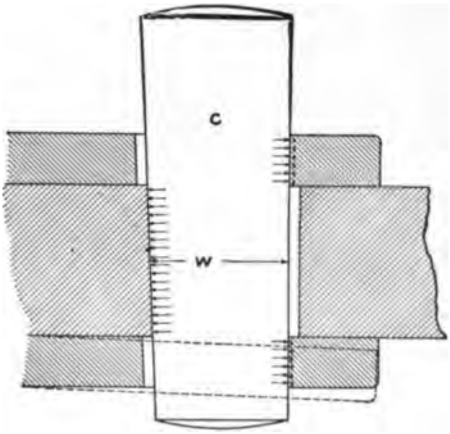


Fig. 74.

In the common use of the cotter for holding the strap at the ends of connecting rods, the strap acts like a modified form of socket. This is shown in Figs. 73 and 74. Here, in addition to holding the strap and rod together lengthwise, it may be necessary to prevent their spreading, and for this purpose an auxiliary piece G with gib ends is used. The tendency without this extra piece is shown by the dotted lines in Fig. 74.

The general mechanical fault with cottered joints is that the action of the load, especially when it constantly reverses, as in pump piston rods, always tends to work the cotter loose. Vibration also tends to produce the same effect. Once this looseness is started in the joint, the cotter loses its pure crushing and shearing action, and begins to partake of the nature of a hammer, and

pounds itself and its bearing surfaces out of their true shape. Instead of a collar on the rod, we often find a taper fit of the rod in the socket; and any looseness in this case is still worse, for the rod then has end play in the socket, and by its "shucking" back and forth tends to split open the socket.

The only answer to these objections is to provide a positive locking device, and take up any looseness the instant it appears.

PROBLEMS ON KEYS, PINS, AND COTTERS.

1. Calculate the safe load in shear which can be carried on a key $\frac{1}{2}$ inch wide, $\frac{3}{8}$ inch thick, and 5 inches long. Assume $S_s = 6{,}000$.

2. Assuming the above key to be $\frac{3}{16}$ inch in hub and $\frac{3}{16}$ inch in shaft, test its proportions for crushing, at $S_c = 16{,}000$.

3. A gear 60 inches in diameter has a load of 3,000 lbs. at the pitch line. The shaft is 4 inches in diameter, in a hub, 5 inches long; and the key is a standard gib key as given in the table. Test its proportions for shearing.

4. A piston rod 2 inches in diameter carries a cotter $\frac{3}{8}$ inch thick, and has an axial load of 20,000 lbs. Calculate the average width of the cotter. $S_s = 9{,}000$.

5. Calculate fiber stress in rod in preceding problem at section through slot.

6. How far from the end of rod must the end of slot be?

7. Calculate the crushing fiber stresses on cotter, rod, and socket.

8. How far from the end of socket must the end of slot be, assuming the socket to be of steel?

BEARINGS, BRACKETS, AND STANDS.

NOTATION—The following notation is used throughout the chapters on Bearings Brackets, and Stands.

A = Area (square inches).
a = Distance between bolt centers (inches).
b = Width of bracket base (inches).
c = Distance of neutral axis from outer fiber (inches).
D = Diameter of shaft (inches).
d = Diameter of bolt body (inches).
d_1 = Diameter at root of thread (inches).
H = Horse-power.
h = Thickness of cap at center (inches).
I = Moment of inertia.
L = Length of bearing (inches).
μ = Coefficient of friction (per cent).
N = Number of revolutions per minute.
n = Number of bolts in cap.
n_1 = Number of bolts in bracket base.
P = Total pressure on bearing (lbs.).
p = Pressure per square inch of projected area (lbs).
S = Safe tensile fiber stress (lbs.).
S_s = " shearing " (lbs.).
T = Total load on bolts at top of bracket (lbs.),
t = Thickness of bracket base (inches).
x = Distance from line of action of load to any section of bracket (inches).

ANALYSIS. Machine surfaces taking weight and pressure of other parts in motion upon them are, in general, known as **bearings.** If the motion is rectilinear, the bearing is termed a **slide, guide,** or **way,** such as the cross slide of a lathe, the cross-head guide of a steam engine, or the ways of a lathe bed.

If the motion is a rotary one, like that of the spindle of a lathe, the simple word "bearing" is generally used.

In any bearing, sliding or rotary, there must be strength to carry the load, stiffness to distribute the pressure evenly over the full bearing surface, low intensity of such pressure to prevent the lubricant from being squeezed out and to minimize the wear, and sufficient radiating surface to carry away the heat generated by friction of the surfaces as fast as it is generated. Sliding bearings are of such varied nature, and exist under conditions so peculiar to each case, that a general analysis is practically impossible beyond that given in the sentence above.

Rotary bearings can be more definitely studied, as there are but two variable dimensions, diameter and length, and it is the proper relation between these two that determines a good bearing. The size of the shaft, as noted under "Shafts," is calculated by taking the bending moment at the center of the bearing, combining it with the twisting moment, and solving for the diameter consistent with the assumed fiber stress. But this size must then be tried for deflection due to the bending load, in order that the requirement for stiffness may be fulfilled. When this is accomplished, the friction at the bearing surface may still generate so much heat that the exposed surface of the bearing will not radiate it as fast as generated, in which case the bearing gets hotter and hotter, until it finally burns out the lubricant and melts the lining of the bearing, and ruin results.

The heat condition is usually the critical one, as it is very easy to make a short bearing which is strong enough and amply stiff for the load it carries, but which nevertheless is a failure as a bearing, because it has so small a radiating surface that it cannot run cool.

The side load which causes the friction and the consequent development of heat, is due to the pull of the belt in the case of pulleys, the load on the teeth of gears, the pull on cranks and

levers, the weight of parts, etc. If we could exert pure torsion on shafts without any side pressure, and counteract all the weight that comes on the shaft, we should not have any trouble with the development of heat in bearings; in fact, there would theoretically be no need of bearings, as the shafts would naturally spin about their axes, and would not need support.

It can be shown, theoretically, that the radiating surface of a bearing increases relatively to the heat generated by a given side load, *only when the length of the bearing is increased.* In other words, increasing the diameter and not the length, theoretically increases the heat generated per unit of time just as much as it increases the radiating surface; hence nothing is gained, and heat accumulates in the bearing as before. This important fact is verified by the design of high-speed bearings, which, it is always noted, are very long in proportion to their diameter, thus giving relatively high radiating power.

Bearings must be rigidly fastened to the body of the machine in some way, and the immediate support is termed a **bracket, frame,** or **housing.** "Bracket" is a very general term, and applies to the supports of other machine parts besides "bearings." It is especially applicable to the more familiar types of bearing supports, and is here introduced to make the analysis complete.

The bracket must be strong enough as a beam to take the side load, the bending moment being figured at such points as are necessary to determine its outline. It may be of solid, box, or ribbed form, the latter being the most economical of material, and usually permitting the simplest pattern. The fastening of the bracket to the main body of the machine must be broad to give stability; the bolts act partly in shear to keep the bracket from sliding along its base, and partly in tension to resist its tendency to rotate about some one of its edges, due to the side pull of the belt, gear tooth, or lever load, as the case may be. The weight of the bracket itself and of the parts it sustains through the bearing, has likewise to be considered; and this acts, in conjunction with the working load on the bearing, to modify the direction and magnitude of the resultant load on the bracket and its fastening.

Stands are forms of brackets, and are subject to the same analysis. The distinction is by no means well defined, although

we usually think more readily of a stand as having an upright or inverted position with reference to the ground. The ordinary "hanger" is a good example of an inverted stand; and the regular "floor stand," found on jack shafts in some power houses, is an example of the general class.

THEORY. As the method of calculation of the diameter of the shaft, as well as its deflection, has been considered under "Shafts," we may assume that the theoretical study of bearings starts on a given basis of shaft diameter D. The main problem then being one of heat control, let us first calculate the amount of heat developed in a bearing by a given side load. The force of friction acts at the circumference of the shaft, and is equal to the coefficient of friction times the normal force; or, for a given side load P, Fig. 75, the force of friction would be μP. The peripheral speed of the shaft for N revolutions per minute is $\frac{\pi DN}{12}$ feet per minute. As work is "force times distance," the work wasted in friction is then $\frac{\mu P \pi DN}{12}$ foot-pounds per minute. One horse-power being equal to 33,000 foot-pounds per minute, we have the equation,

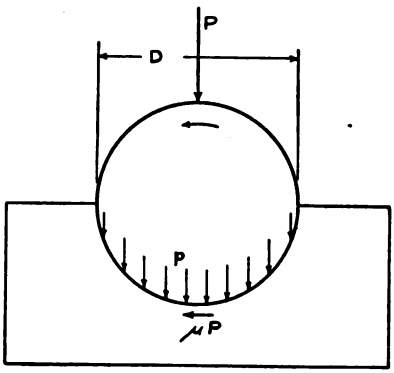


Fig. 75.

$$H = \frac{\mu P \pi DN}{12 \times 33{,}000}. \qquad (113)$$

The value of μ for ordinary, well-lubricated bearings, may run as low as 5 per cent; but as the lubrication is often impaired, it quite commonly rises to 10 or 12 per cent. A value of 8 per cent is a fair average. This amount of horse-power is dissipated through the bearing in the form of heat. If we could exactly determine the ability that each particle of the metal around the shaft had to transmit the heat, or to pass it along to the outside of the casting, and if we could then determine the ability of the particles of air surrounding the casting to receive and carry away

this heat, we could calculate just such proportions of the bearing and its casing as would never choke or retard this free transfer of heat away from the running surface.

Such refined theory is not practical, owing to the complicated shapes and conditions surrounding the bearing. The best that we can do is to say that for the usual proportions of bearings the side load may exist up to a certain intensity of "pressure per square inch of projected area" of bearing, or, in form of an equation,

$$P = pLD. \qquad (114)$$

The constant p is of a variable nature, depending on lubrication, speed, air contact, and other special conditions. For ordinary bearings having continuous pressure in one direction, and only fair lubrication, 400 to 500 is an average value. When the pressure changes direction at every half-revolution, the lubricant has a better chance to work fully over the bearing surface, and a higher value is permissible, say, 500 to 800. In locations where mere oscillation takes place, not continuous rotation, and reversal of pressure occurs, as on the cross-head pin of a steam engine, p may run as high as 900 to 1,200. On the crank pins of locomotives, which have the reversal of pressure, and the benefit of high velocity through the air to facilitate cooling, the pressures may run equally high. On the eccentric crank pins of punching and shearing machines, where the pressure acts only for a brief instant and at intervals, the pressure ranges still higher without any dangerous heating action.

When a bearing, for practical reasons, is provided with a cap held in place by bolts or studs, the *theory* of the cap and bolts is of little importance, unless the load comes directly against the cap and bolts. Except in the latter case, the proportions of the cap and the size of the bolts are dependent upon general appearance and utility, it being manifestly desirable to provide a substantial design, even though some excess of strength is thereby introduced.

For the worst case of loading, however, which is when the cap is acted upon by the direct load, such as P in Fig. 76, we have the condition of a centrally loaded beam supported at the bolts. It is probable that the beam is partially fixed at the ends by the clamping of the nut; also that the load P, instead of being con-

centrated at the center, is to some extent distributed. It is hardly fair to assume the external moment equal to $\frac{Pa}{8}$ or $\frac{Pa}{4}$, the one being too small, perhaps, and the other too large. It will be reasonable to take the external moment at $\frac{Pa}{6}$, in which case, equat-

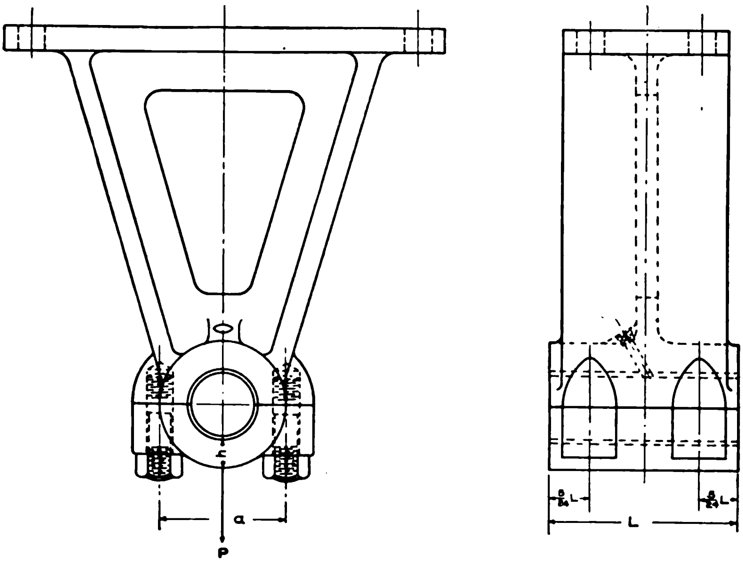


Fig. 76.

ing the external moment to the internal moment of resistance,

$$\frac{Pa}{6} = \frac{SI}{c} = \frac{SLh^2}{6}, \qquad (115)$$

from which, the length of bearing being known, we may calculate the thickness h.

One bolt on each side is sufficient for bearings not more than 6 inches long, but for longer bearings we usually find two bolts on a side. The theoretical location for two bolts on a side, in order that the bearing may be equally strong at the bolts and at the center of the length, may be shown by the principles of mechanics to be $\frac{5}{24}$ L from each end, as indicated in Fig. 76.

The bolts are evidently in direct tension, and if equally loaded

would each take their fractional share of the whole load P. This is difficult to guarantee, and it is safer to consider that $\frac{2}{3}$ P may be taken by the bolts on one side. On this basis, for total number of bolts n, equating the external force to the internal resistance of the bolts, we have :

$$\frac{2}{3}P = \frac{S\pi d_1^2}{4} \times \frac{n}{2}, \qquad (116)$$

from which the proper commercial diameter may be readily found.

The bracket may have the shape shown in Fig. 77. The portion at B is under direct shearing stress; and if A be the area at this point, and S_s the safe shearing stress, then, equating the external force to the internal shearing resistance,

$$P = AS_s. \qquad (117)$$

The same shear comes on all parts of the bracket to the left of the load, but there is an excess of shearing strength at these points.

At the point of fastening, the bolts are in shear, due to the same load, for which the equation is

$$P = \frac{\pi d^2}{4} n_1 S_s. \qquad (118)$$

For the upper bolts, the case is that of direct tension, assuming that the whole bracket tends to rotate about the lower edge E. To find the load T on these bolts, we should take moments about the point E, as follows:

$$PL_1 = Tl; \text{ or } T = \frac{PL_1}{l}. \qquad (119)$$

Then, equating the external force to the internal resistance,

$$T = \frac{PL_1}{l} = \frac{\pi d_1^2}{4} \times \frac{n_1}{2} S. \qquad (120)$$

The upper flange is loaded with the bolt load T, and tends to break off at the point of connection to the main body of the bracket, the external moment, therefore, being Tr. The section of the flange is rectangular; hence the equation of external and internal moments is:

$$\mathrm{T}r = \frac{\mathrm{PL}_1}{l} r = \frac{\mathrm{S}bt^2}{6}. \qquad (121)$$

It may be noted that the lower bolts act on such a small leverage about E, that they would stretch and thus permit all the load to be thrown on the upper bolts; this is the reason why they are not subject to calculation for tension.

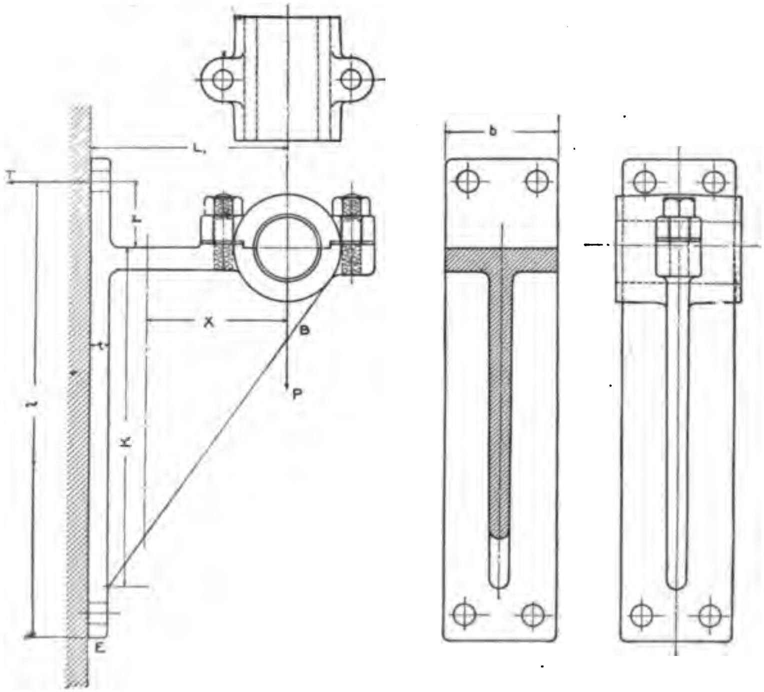


Fig. 77.

The section of the bracket to the left of the load P is dependent upon the bending moment, for, if this section is large enough to take the bending moment properly, the shear may be disregarded. It should be calculated at several points, to make sure that the fiber stress is within allowable limits. The general expression for the equation of moments is, for any section at leverage x,

$$\mathrm{P}x = \frac{\mathrm{SI}}{c}, \qquad (122)$$

from which, by the proper substitution of the

ertia of the section, the fiber stress can be calculated. The moment of inertia for simple ribbed sections can be found in most handbooks. The process of solution of the above equation, though simple, is apt to be tedious, and is not considered necessary to illustrate here.

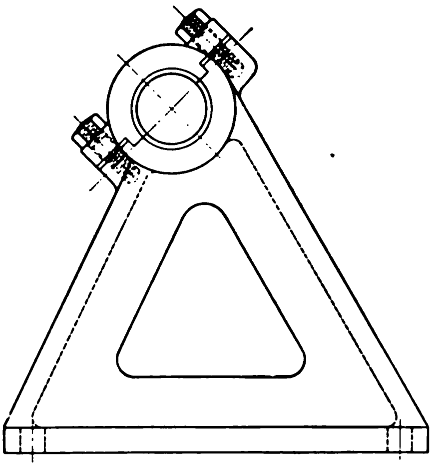


Fig. 78.

PRACTICAL MODIFICATION. Adjustment is an important practical feature of bearings. Unless the proportions are so ample that wear is inappreciable, simple and ready adjustment must be provided. The taper bushing, Fig. 79, is neat and satisfactory for machinery in which expense and refinement are permissible. This is true of some machine tools, but is not true of the general "run" of bearings. The most common form of adjustment is secured by the plain cap (which may or may

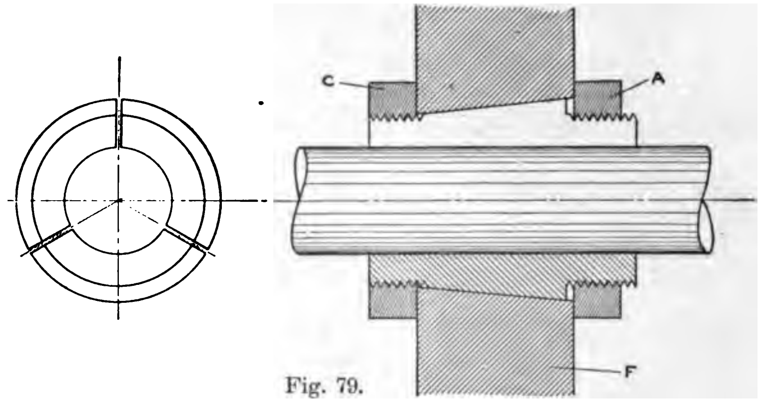


Fig. 79.

not be tongued into the bracket), with liners placed in the joint when new, which may subsequently be removed or reduced so as to allow the cap to close down upon the shaft. Several forms of cap bearings are illustrated in Figs. 80, 81, and 82.

Large engine shaft bearings have special forms of adjustment by means of wedges and screws, which take up the wear in all directions, at the same time accurately preserving the alignment of the shafts; but this refinement is seldom required for shafts of ordinary machinery.

In cases where the cap bearing is not applicable, a simple bushing may be used. This may be removed when worn, and a

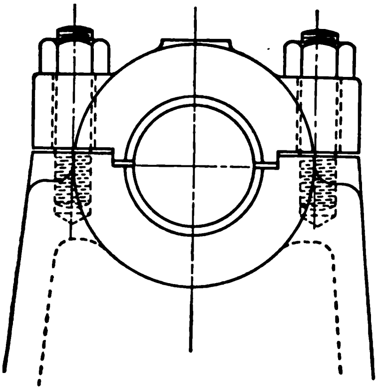


Fig. 80.

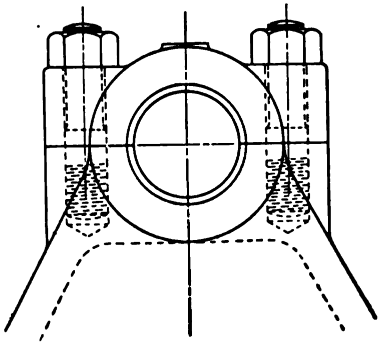


Fig. 81.

new one inserted, the exact alignment being maintained, as the outside will be concentric with the original axis of shaft, regardless of the wear which has taken place in the bore.

The lubrication of bearings is a part of the design, in that the lubricant should be introduced at the proper point, and pains taken to guarantee its distribution to all points of the running surface. The method of lubrication should be so certain that no excuse for its failure would be possible. Grease is a successful lubricator for heavy loads and slow speeds, oil for light loads and high speeds.

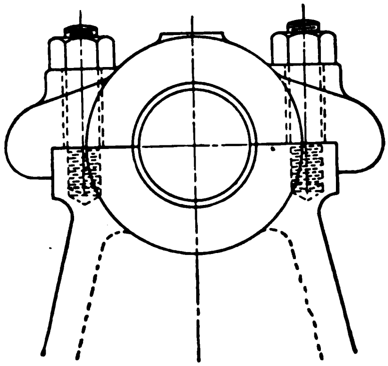


Fig. 82.

In order to insure the lubricant reaching the sliding surfaces and entering between them, it must be introduced at a point

where the pressure is moderate, and where the motion of the parts will naturally lead it to all points of the bearing. Grooves and channels of ample size assist in this regard. A special form of bearing uses a ring riding on the shaft to carry the oil constantly from a small reservoir beneath the shaft up to the top, where it is distributed along the bearing and finally flows back to the reservoir and is used again.

The materials of which bearings are made vary with the service required and with the refinement of the bearing. Cast iron makes an excellent bearing for light loads and slow speeds, but it is very apt to "seize" the shaft in case the lubrication is in the least degree impaired. Bronze, in its many forms of density and hardness, is extensively used for high-grade bearings, but it also has little natural lubricating power, and requires careful attention to keep it in good condition.

Babbitt, a composition metal, of varying degrees of hardness, is the most universal and satisfactory material for ordinary bearings. It affords a cheap method of production, being poured in molten form around a mandrel, and firmly retained in its casing or shell through dovetailed pockets into which the metal flows and hardens. It requires no boring or extensive fitting. Some scraping to uniform bearing is necessary in most cases, but this is easily and cheaply done. Babbitt is a durable material, and has some natural lubricating power, so that it has less tendency to heat with scanty lubrication than any of the materials previously mentioned. Almost any grade of bearing may be produced with babbitt. In its finest form the babbitt is hammered, or pened, into the shell of the bearing, and then bored out nearly to size, a slightly tapered mandrel being subsequently drawn through, compressing the babbitt and giving a polished surface.

A combination bearing of babbitt and bronze is sometimes used. In this the bronze lies in strips from end to end of the bearing, and the babbitt fills in between the strips. The shell, being of bronze, gives the required stiffness, and the babbitt the favorable running quality.

PROBLEMS ON BEARINGS, BRACKETS, AND STANDS.

1. The allowable pressure on a bearing is 300 pounds per

square inch of projected area. What is the required length of the bearing if the total load is 4,500 pounds and the diameter is 3 inches?

2. The cross-head pin of a steam engine must be 2.5 inches in diameter to withstand the shearing strain. If the maximum pressure is 10,000 pounds, what length should be given to the pin?

3. The journals on the tender of a locomotive are $3\frac{1}{2} \times 7$ inches. The total weight of the tender and load is 60,000 pounds. If there are 8 journals, what is the pressure per square inch of projected area?

4. What horse-power is lost in friction at the circumference of a 3-inch bearing carrying a load of 6,000 pounds, if the number of revolutions per minute is 150 and the coefficient of friction is assumed to be 5 per cent?

5. The cast-iron bracket in Fig. 77 has a load P of 1,000 pounds. Determine the fiber stress in the web section at the base of the bracket if the thickness is taken at $\frac{1}{2}$ inch, and $L_1 = 12$ inches; $l = 20$ inches; $k = 11$ inches; $t = 1$ inch.

6. Calculate the diameter of the bolts at the top of the bracket.

7. Assuming r equal to 6 inches, what is the fiber stress at the root of flange?

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